Phase transition in a static granular system

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Abstract

We have measured the force needed by a rod to penetrate a static column of grains for different well-defined particle volume fractions ϕ. Measurements on our system reveal a phase transition at $\phi = 0.590 \pm 0.003$.

It was first noted long ago, at least as far back as the investigations of Reynolds (1885, 1886), that there are distinct fluid and solid states of static granules such as sand. The distinction between fluid and solid for static states is based on the degree of resistance to shear, e.g. to motion of an inserted rod: if a hard rod is easily inserted and/or withdrawn, the material is acting as a fluid, while if it difficult to insert and/or withdraw, the material is acting as a solid. There was even a well-known magic trick a century ago based on this change of state (Branson, 1922; Aste & Weaire, 2000). A pot with a narrow neck is filled with rice and a knife is repeatedly shown to be easily inserted and removed. Then the knife is inserted and the pot is shaken (compactifying the rice). The whole pot can then be lifted by the knife handle and swung around the performer's head.

While the existence of fluid and solid phases has been widely discussed, a sharp distinction between the two phases has remained elusive (Jaeger *et al.*, 1996; Kadanoff, 1999; de Gennes, 1999). The distinction between different states of a granular system is hampered by the difficulty in preparing a well-defined initial state (de Gennes, 1999). The effort to overcome this was advanced significantly by Nowak *et al.* (1998), who used a mechanical tapping protocol to obtain well-defined densities in the range 0.628 to 0.658. Recently Schröter *et al.* (2005) showed that a different protocol, based on expanding the granular medium by pulses of fluid from below, could be used to prepare a column of grains with a well-defined particle volume fraction. The volume fraction was obtained to 0.1% in a broad range, 0.566 to 0.609. Using this technique, we have found a phase transition at density 0.590.

Experiment. Measurements are performed with a home built granular penetrometer: A translation stage driven by a stepper motor with a step size of 2.5 μm moves a stainless steel rod with a diameter of 6.3 mm downwards into a granular sample. The force needed for penetration is measured with a load cell with a full range of 10 N (Honeywell, Model 31).

The sample consists of soda lime glass beads from Cataphote with a diameter of 265 ± 15 μm and a density of 2.484 ± 0.002 g/cm^3 (measured with a Micromeritics gas pycnometer AccuPhys 1330).

The beads are contained in a water-fluidized bed where flow pulses of different flow rates allow us to control their volume fraction (Schröter *et al.*, 2005). The beads are fluidized inside a square bore glass tube (39.9×39.9 mm^2). The ratio of inner tube diameter to rod diameter is 6.3, larger than the value of 5 that Stone *et al.* (2004) found to be the ratio where the influence of the vessel diameter vanishes.

Flow pulses are generated using a digital gear pump (Barnant Co., model no. 75211). The volume fraction is determined from bed height measurements with a CCD camera at a resolution of 39.6 μm/pixel.

Results. Figure 1 shows that the force on the penetrating rod increases monotonically with its depth; however, the slope of the curve depends strongly on the volume fraction of the sample. The dependence of the

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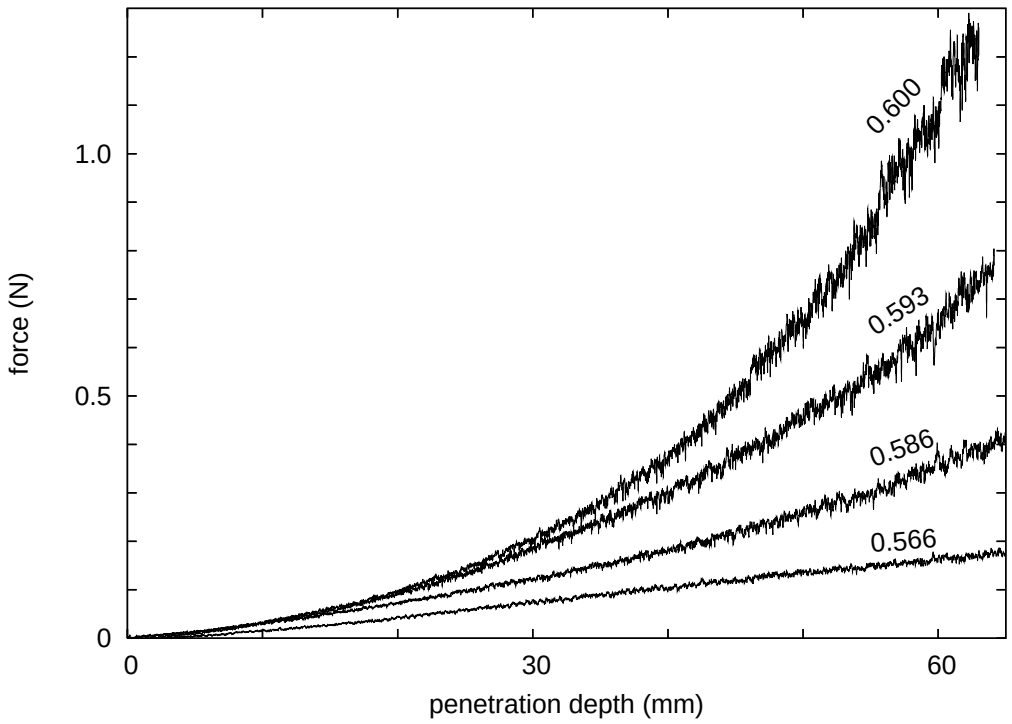

Figure 1: Penetration force as a function of depth for different volume fractions. All curves were measured with a penetration speed of 10 mm/s. The total sample height was 110 mm (at $\phi = 0.6$).

force on volume fraction at two different depths is plotted in Fig. 2. There is a clear change in the slopes ast $\phi = 0.59$. This indicates a qualitative change in the behavior of the sample. The figure also shows that the penetration force is independent of the speed of the rod. The absence of a dependence on penetration speed is in agreement with the results of Stone *et al.* (2004) at $\phi = 0.59$. It also shows that the flow of water induced by the penetrating rod does not alter the results.

Discussion. It is a defining characteristic of granular systems that the grains are heavy enough for gravity to play a significant role. Nontheless it is tempting to compare the behavior of static granular systems with either the classical statistical mechanics model system of hard spheres, or colloidal systems designed to be so modeled. The latter consists of particles suspended in a liquid and treated to reduce the interaction between the particles to approximate bodies which do not interact except to prevent interpenetration (Pusey & Megen, 1986). The colloidal system can be prepared at different densities by use of a centrifuge. The states of the system have been found to be characterized by a freezing density $\phi_f = 0.407$ and a melting density $\phi_m = 0.442$, such that (as determined by diffraction) for densities below ϕ_f the system is a fluid; for densities above ϕ_m it is a crystalline solid; and for densities between ϕ_f and ϕ_m the system is a mixture of the two states (Pusey & Megen, 1986). This phase behavior (but with $\phi_f = 0.494$ and $\phi_m = 0.545$) was found earlier for a model system of hard spheres by Monte Carlo and molecular dynamics simulations; see Alder & Hoover (1968) for a survey on these early simulations.

There are significant differences between a colloidal system and our static granular system, most notably that the colloid particles are in thermal equilibrium while the collection of granules is not – it is in fact static. Furthermore, in the colloidal system the pure solid and fluid phases are separated by an interval of densities representing coexistence, with a sharp transition at each end, while for our granular system we observe only one transition. However, the existence of sharp transitions, indicating well-defined distinct phases, is a striking similarity between the colloidal and granular systems, and supports the effort to find a theory for granular systems similar to that used for these colloidal systems, that is, some modification of equilibrium ensemble theory.

Another significant difference between the two systems is that the transition in the colloidal system cor-

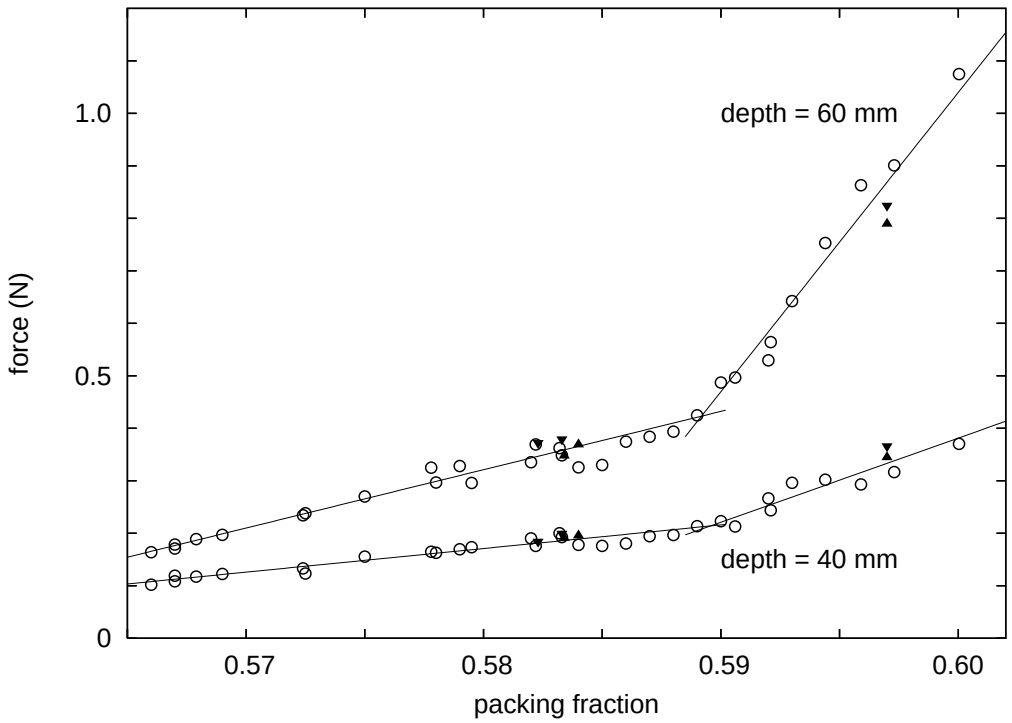

Figure 2: The dependence of the penetration force on volume fraction changes at ϕ =0.590. Data represented by ∘ were measured at a penetration speed of 10 mm/s, ▼ at 5 mm/min and ▲ at 20 mm/min. The straight lines are least-square fits to the forces above and below $\phi = 0.59$.

responds to a change of symmetry between homogeneous and crystalline, while the transition in the granular system is marked by a qualitative change in the resistance to shear. However, a recent consideration of the transition in the equilibrium hard sphere model (Bowen *et al.*, 2006) suggests that the symmetry change may be hiding the fundamental mechanism in that model, a type of geometric constraint, which may also underlie the phases of the granular system.

The density at which we find the transition in our granular system coincides with earlier findings of interesting behavior: Schröter *et al.* (2005) measured volume fluctuations around a static steady state using a series of identical flow pulses in a fluidized bed, and those fluctuations were found to exhibit a parabolic minimum at volume fractions between 0.587 and 0.596, depending on the surface roughness of the beads. Arguments based on the central limit theorem showed that this minimum corresponds to a minimal number of beads being contained in a statistically independent region, which is tantamount to a minimum in the correlation length. In another study Goldman & Swinney (2006) found that if a fluidized bed is slowly defluidized, it exhibits a behavior similar to that of a supercooled liquid, with a glass transition at ϕ =0.582 and an arrest transition at $\phi = 0.594$. Near the glass transition the system exhibited dynamical heterogeneities with a spatial correlation length of several particle diameters, while at the arrest transition ϕ became nearly independent of the flow rate and the correlation length went to zero.

In conclusion, we have taken advantage of the recent discovery of a method to produce history-independent columns of granules with well-defined packing fractions in the range 0.57 to 0.61 (Schröter *et al.*, 2005) to search for a phase transition as a function of packing fraction. The observation of this transition suggests the appropriateness of an ensemble theory (de Gennes, 1999).

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