Solutions of nonstationary Dirac equation with the potential slowly depending on time *

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25 апреля 2002 г.

Asymptotic behaviour of the solution of the Cauchy problem for the nonstationary Dirac equation with the potential slowly depending on time is studied. Construction of asymptotic solution is based on spectral decomposition of the solution in current moment of time. It doesn't use the adiabatic theorem in scattering theory.

1 Introduction

The objective of this paper is the investigation of the solutions of nonstationary Dirac equation

$$-i\Phi_t = \mathrm{H}(\varepsilon t)\Phi, \quad \varepsilon \ll 1 \tag{1}$$

with operator H which slowly depend on time. The asymptotic formulas for the operator H with discrete spectrum are well known. Construction of asymptotic formulas for the solution of the equation (1) then operator H has continuous spectrum are usually based on adiabatic theorem in scattering theory (see [1]). Application of this method require decomposition of operator H in following sum

$$\mathrm{H}(\varepsilon t) = \mathrm{H}_0 + \mathrm{V}(\varepsilon t), \tag{2}$$

where operator H_0 doesn't depend on t .In particular main term of this such solution is completely determined by value of operator H at $t = 0$. Constructions of this method is closely connected with scattering theory for nonstationary equation (1)(see [2] and [3]). We use another method that doesn't depend on the adiabatic theorem in scattering theory.We consider (as a suitable model) one-dimensional Dirac operator on the whole line with rapidly decaying potential

$$\mathrm{H(t)} = \mathrm{i}\sigma_3 \frac{\mathrm{d}}{\mathrm{dx}} + \begin{pmatrix} 0 & p \\ \overline{p} & 0 \end{pmatrix}. \tag{3}$$

At this case operator H has continuous spectrum on the whole line and it has not discrete spectrum. Construction of asymptotic solution is based on spectral decomposition of the solution in current moment of time.

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*This paper was supported by RFFI grant 02-01-00798

In §2 we consider main information about spectral theory for the stationary Dirac operator. In $3 formal solution for nonstationary Dirac equation is constructed .In $4 we prove that formal solution is asymptotic expansion for precise solution of the Cauchy problem for the nonstationary Dirac equation.Finally, in $5 we compare our asymptotic formulas with result that are based on the adiabatic theorem in scattering theory.

2 Spectral theory for Dirac operator

Let us describe some information about spectral theory for Dirac operator (see [4]). Consider Dirac operator on the whole line with rapidly decaying potential

$$\mathrm{H} = \mathrm{i}\sigma_3 \frac{\mathrm{d}}{\mathrm{dx}} + \begin{pmatrix} 0 & p(x) \\ \overline{p(x)} & 0 \end{pmatrix}, p \in S(-\infty, +\infty).$$

Here $\mathrm{S}(-\infty, +\infty)$ - Schwartz class. Let us fix solutions $\Psi(x)$ and $\tilde{\Psi}(x)$ of spectral equation

$$\mathrm{H}\Phi = \lambda\Phi, \quad \Phi \in \mathrm{Mat}(2 \times 2), \tag{4}$$

which satisfy conditions

$$\Psi(x, \lambda) = \exp(-i\sigma_3\lambda x) + o(1), x \to +\infty,$$

$$\tilde{\Psi}(x, \lambda) = \exp(-i\sigma_3\lambda x) + o(1), x \to -\infty.$$

Lemma 1. *The solutions $\Psi(x)$ and $\tilde{\Psi}(x)$ exist and are unique. They are infinitely differentiable functions of x and λ and have following asymptotics*

$$\Psi(x, \lambda) = \exp(-i\sigma_3\lambda x) \begin{pmatrix} a(\lambda) & \overline{b(\lambda)} \\ b(\lambda) & \overline{a(\lambda)} \end{pmatrix} + o(1), \quad x \to -\infty,$$

$$\tilde{\Psi}(x, \lambda) = \exp(-i\sigma_3\lambda x) \begin{pmatrix} \overline{a(\lambda)} & -\overline{b(\lambda)} \\ -b(\lambda) & a(\lambda) \end{pmatrix} + o(1), \quad x \to +\infty.$$

The coefficients a and b satisfy following conditions

$$a(\lambda) = 1 + \tilde{a}(\lambda); \ \tilde{a}(\lambda), b(\lambda) \in S(-\infty, +\infty),$$

$$|a|^2 = |b|^2 + 1.$$

The proof of this lemma follows from the analysis of the corresponding Volterra integral equations.

Consider $\mathbf{\Psi}_1, \mathbf{\Psi}_2$ and $\tilde{\mathbf{\Psi}}_1, \tilde{\mathbf{\Psi}}_2$ are vector-functions which are column vectors of matrix Ψ and $\tilde{\Psi}$

$$\Psi = (\mathbf{\Psi}_1, \mathbf{\Psi}_2), \tilde{\Psi} = (\tilde{\mathbf{\Psi}}_1, \tilde{\mathbf{\Psi}}_2).$$

Operator H has continuous spectrum on the whole line. We can consider vector-functions $\mathbf{f}$ and $\mathbf{g}$,

$$\mathbf{f} = \frac{\tilde{\mathbf{\Psi}}_1}{\overline{a}}, \quad \mathbf{g} = \frac{\mathbf{\Psi}_2}{\overline{a}}$$

(see[4]) as eigen-functions of the continuous spectrum for operator H. In particular

$$\int_{-\infty}^{\infty} d\lambda (\mathbf{f}(x,\lambda)\overline{\mathbf{f}^{\top}(y,\lambda)} + \mathbf{g}(x,\lambda)\overline{\mathbf{g}^{\top}(y,\lambda)}) = 2I\pi\delta(x-y),$$

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

Here $\delta(x)$ is delta-function. Functions $\mathbf{f}$ and $\mathbf{g}$ have following asymptotic properties:

$$\mathbf{f}(x,\lambda,\tau) = \exp(-i\sigma_3\lambda x)\begin{pmatrix} 1/\overline{a} \\ 0 \end{pmatrix} + \mathbf{o}(1), \quad x \to -\infty, \tag{5}$$

$$\mathbf{f}(x,\lambda,\tau) = \exp(-i\sigma_3\lambda x)\begin{pmatrix} 1 \\ b/\overline{a} \end{pmatrix} + \mathbf{o}(1), \quad x \to +\infty, \tag{6}$$

$$\mathbf{g}(x,\lambda,\tau) = \exp(-i\sigma_3\lambda x)\begin{pmatrix} \overline{b}/\overline{a} \\ 1 \end{pmatrix} + \mathbf{o}(1), \quad x \to -\infty, \tag{7}$$

$$\mathbf{g}(x,\lambda,\tau) = \exp(-i\sigma_3\lambda x)\begin{pmatrix} 0 \\ 1/\overline{a} \end{pmatrix} + \mathbf{o}(1), \quad x \to +\infty, \tag{8}$$

Let us consider space of vector-functions S_0

$$\mathbf{c} \in S_0 \longleftrightarrow \mathbf{c} = \begin{pmatrix} c_1 \\ c_2 \end{pmatrix}; c_1, c_2 \in S(-\infty, +\infty).$$

Transformation F_0 which are fixed by functions $\mathbf{f}$ and $\mathbf{g}$ on the space S_0

$$\begin{pmatrix} d_1(\lambda) \\ d_2(\lambda) \end{pmatrix} = F_0 \begin{pmatrix} c_1(x) \\ c_2(x) \end{pmatrix} = \int_{-\infty}^{\infty} dx \begin{pmatrix} \overline{\mathbf{f}^{\top}(x,\lambda)} \\ \overline{\mathbf{g}^{\top}(x,\lambda)} \end{pmatrix} \begin{pmatrix} c_1(x) \\ c_2(x) \end{pmatrix},$$

(here $\mathbf{a}^{\top}$-denote transposition of vector $\mathbf{a}$)has properties that are analogous to properties of the Fourier transformation. In particular, we have following formulae for the inverse transformation

$$\begin{pmatrix} c_1(x) \\ c_2(x) \end{pmatrix} = F_0^{-1} \begin{pmatrix} d_1(\lambda) \\ d_2(\lambda) \end{pmatrix} = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\lambda (\mathbf{f}(x,\lambda), \mathbf{g}(x,\lambda)) \begin{pmatrix} d_1(\lambda) \\ d_2(\lambda) \end{pmatrix}. \tag{9}$$

We can find general solution of nonhomogeneous equation

$$\mathrm{H}\mathbf{v} - \lambda\mathbf{v} = \mathbf{h}(x), \quad \mathbf{h} \in S(-\infty, \infty)$$

in the terms of functions $\mathbf{f}$ and $\mathbf{g}$

$$\mathbf{v}(x) = \int_0^x dy K_\lambda(x,y)\mathbf{h}(y) + d_1\mathbf{f} + d_2\mathbf{g}$$

Here d_1 and d_2 are arbitrary constant coefficients, $K_\lambda(x,y)$ is matrix 2x2

$$K_\lambda(x,y) = \left\{ \begin{array}{ll} \mathbf{g}(x)(\tilde{\psi}_{21}(y), \tilde{\psi}_{11}(y)), & x > 0 \\ \mathbf{f}(x)(\psi_{22}(y), \psi_{12}(y)), & x < 0 \end{array} \right. ,$$

where ψ_{ij} and $\tilde{\psi}_{ij}$ are elements of matrix Ψ and $\tilde{\Psi}$.

3 Formal solution of nonstationary Dirac equation.

Consider solution of the Cauchy problem for nonstationary Dirac equation

$$-i\mathbf{\Phi}_t = \mathrm{H}(\tau)\mathbf{\Phi}, \ \ \tau = \varepsilon t, \ \varepsilon \ll 1 \tag{10}$$

$$\mathbf{\Phi}|_{t=o} = \phi_0(x).$$

Here $\mathbf{\Phi}(x,t)$ is vector-function with two components, $\mathrm{H}(\tau)$ is Dirac operator

$$\mathrm{H}(\tau) = \mathrm{i}\sigma_3\frac{\mathrm{d}}{\mathrm{dx}} + \left(\begin{array}{cc} 0 & p(x,\tau) \\ \overline{p(x,\tau)} & 0 \end{array} \right), \ \tau = \varepsilon t,$$

$$p(x,\tau) \in C^\infty((-\infty,+\infty)\times[0,\tau_0]),$$

$$\phi_0(x) \in S_0, \ \ \forall \ \ l \geq 0 \ \ \partial_\tau^l p(\bullet,\tau) \in S(-\infty,+\infty),$$

σ_3 is Pauli matrix. We construct formal solution of the Cauchy problem (10) for $\tau \in [0,\tau_0]$ with the help of following decomposition

$$\hat{\mathbf{\Phi}}(x,t) = \int_{-\infty}^{\infty} d\lambda \exp(i\lambda t)\sum_{l=0}^{\infty}\varepsilon^l\mathbf{F}_l(x,\lambda,\tau). \tag{11}$$

Theorem 1. *The Cauchy problem (10) has single formal solution of the type (11) with following properties:*

$$\mathbf{F}_l(x,\lambda,\tau) \in C^\infty((-\infty,+\infty)\times(-\infty,+\infty)\times[0,\tau_0]), \ \ \partial_\tau\mathbf{F}_l(x,\bullet,\tau) \in S_0, \forall l \geq 0$$

$$\mathbf{F}_l = \exp(-i\sigma_3\lambda x)\left(\begin{array}{c} P_l(x,\lambda,\tau) \\ p_l^0(\lambda) \end{array} \right) + \mathbf{o}(1), \ \ x \to -\infty, \tag{12}$$

$$\mathbf{F}_l = \exp(-i\sigma_3\lambda x)\left(\begin{array}{c} q_l^0(\lambda) \\ Q_l(x,\lambda,\tau) \end{array} \right) + \mathbf{o}(1), \ \ x \to +\infty. \tag{13}$$

Here P_l and Q_l are polynomials in x

$$P_l = \sum_{m=0}^{l} p_{lm}(\tau,\lambda)x^m, \quad Q_l = \sum_{m=0}^{l} q_{lm}(\tau,\lambda)x^m,$$

with smooth coefficients

$$\partial_\tau^l p_{lm}(\bullet,\tau) \in S(-\infty,+\infty), \partial_\tau^l q_{lm}(\bullet,\tau) \in S(-\infty,+\infty),$$

$p_l^0(\lambda)$ *and* $q_l^0(\lambda)$ *are functions from Schwarz class (they don't depend on* τ *and* x *).*

Let us substitute series (11) in equation (10). We obtain a system of recurrent relations for the coefficients of the series

$$-i(\mathbf{F}_l)_\tau = (\mathrm{H}(\tau)-\lambda)\mathbf{F}_{l+1}, l = 0,1,2...$$

This implies that

$$(\mathrm{H}(\tau)-\lambda)\mathbf{F}_0(x,\lambda,\tau) = 0.$$

Thus function $\mathbf{F}_0(x,\lambda,\tau)$ is the solution of stationary equation (4). Arbitrary solution of the stationary equation has a form

$$\mathbf{F}_0(x,\lambda,\tau) = d_1^0(\lambda,\tau)\mathbf{f}(x,\lambda,\tau) + d_2^0(\lambda,\tau)\mathbf{g}(x,\lambda,\tau).$$

Let us use asymptotic properties of the solutions $\mathbf{f}(x,\lambda,\tau)$ and $\mathbf{g}(x,\lambda,\tau)$ (5)-(8). We obtain

$$\mathbf{F}_0(x,\lambda,\tau) = \exp(-i\sigma_3\lambda x)\left[d_1^0\begin{pmatrix} 1/\overline{a} \\ 0 \end{pmatrix} + d_2^0\begin{pmatrix} \overline{b}/\overline{a} \\ 1 \end{pmatrix} + \mathbf{o}(1)\right], \quad x \to -\infty,$$

$$\mathbf{F}_0(x,\lambda,\tau) = \exp(-i\sigma_3\lambda x)\left[d_1^0\begin{pmatrix} 1 \\ b/\overline{a} \end{pmatrix} + d_2^0\begin{pmatrix} 0 \\ 1/\overline{a} \end{pmatrix} + \mathbf{o}(1)\right], \quad x \to +\infty.$$

This implies that ,

$$(d_1^0)_\tau = 0, \quad (d_2^0)_\tau = 0$$

(see (12) and (13)). From initial data of Cauchy problem (10) we have

$$\begin{pmatrix} d_1^0 \\ d_2^0 \end{pmatrix} = \frac{1}{2\pi}\int_{-\infty}^{+\infty} dx \begin{pmatrix} \overline{\mathbf{f}^\top(x,\lambda,0)} \\ \overline{\mathbf{g}^\top(x,\lambda,0)} \end{pmatrix} \phi_0(x). \tag{14}$$

Let us use the induction on l for further proof of theorem. Let statement of theorem is true for all $l \le N$. In particular we have

$$\frac{\partial \mathbf{F}_N(y,\lambda,\tau)}{\partial \tau} = \exp(-i\sigma_3\lambda x)\begin{pmatrix} \tilde{P}_N(x,\lambda,\tau) \\ 0 \end{pmatrix} + \mathbf{o}(1), \quad x \to -\infty, \tag{15}$$

$$\frac{\partial \mathbf{F}_N(y,\lambda,\tau)}{\partial \tau} = \exp(-i\sigma_3\lambda x)\begin{pmatrix} 0 \\ \tilde{Q}_N(x,\lambda,\tau) \end{pmatrix} + \mathbf{o}(1), \quad x \to +\infty. \tag{16}$$

Here

$$\tilde{P}_l(x,\lambda,\tau)=\frac{\partial P_l}{\partial\tau},\ \ \tilde{Q}_l(x,\lambda,\tau)=\frac{\partial Q_l}{\partial\tau}.$$

For $l=N+1$ we have

$$\mathbf{F}_{N+1}(x,\lambda,\tau)=-i\int_0^x dyK_\lambda(x,y,\tau)\frac{\partial\mathbf{F}_N(y,\lambda,\tau)}{\partial\tau}+ \tag{17}$$

$$+d_1^{N+1}(\lambda,\tau)\mathbf{f}(x,\lambda,\tau)+d_2^{N+1}(\lambda,\tau)\mathbf{g}(x,\lambda,\tau).$$

Here $K_\lambda(x,y,\tau)$ is matrix function. It is constructed with the help of functions $\mathbf{f}$ and $\mathbf{g}$

$$K_\lambda(x,y,\tau)=\left\{\begin{array}{ll}\mathbf{g}(x,\lambda,\tau)(\tilde{\psi}_{21}(y,\lambda,\tau),\tilde{\psi}_{11}(y,\lambda,\tau)), & x>0\\ \mathbf{f}(x,\lambda,\tau)(\psi_{22}(y,\lambda,\tau),\psi_{12}(y,\lambda,\tau)), & x<0\end{array}\right.$$

Consider asymptotic behaviour of the function $\mathbf{F}_{N+1}(x,\lambda,\tau)$ as $x\to-\infty$. Let us substitute asymptotic formulas (5) and (7) for the functions $\mathbf{f}(x,\lambda,\tau)$ and $\mathbf{g}(x,\lambda,\tau)$ and asymptotics for derivative of the function $\mathbf{F}_N$ (15) in relation (17). Integral in the right part of relation (17) has following asymptotic behaviour

$$-i\int_0^x dyK_\lambda(x,y,\tau)\frac{\partial\mathbf{F}_N(y,\lambda,\tau)}{\partial\tau}=\exp(-i\sigma_3\lambda x)\begin{pmatrix}\hat{P}_{N+1}(x,\lambda,\tau)\\ \hat{p}_{N+1}(\lambda,\tau)\end{pmatrix}+\mathbf{o}(1),$$

as $x\to-\infty$, where $\hat{P}_{N+1}(x,\lambda,\tau)$ is appropriate polynomial in x (it has degree $N+1$) and $\hat{p}_{N+1}(\lambda,\tau)$ is coefficient which does not depend on x.

$$\mathbf{F}_{N+1}(x,\lambda,\tau)=\exp(-i\sigma_3\lambda x)\left[\begin{pmatrix}\hat{P}_{N+1}(x,\lambda,\tau)\\ \hat{p}_{N+1}(\lambda,\tau)\end{pmatrix}+\right.$$

$$\left.+\,d_1^{N+1}\begin{pmatrix}1/\overline{a}\\ 0\end{pmatrix}+d_2^{N+1}\begin{pmatrix}\overline{b}/\overline{a}\\ 1\end{pmatrix}\right]+\mathbf{o}(1),\ \ x\to-\infty.$$

Taking into consideration the relation (12) we receive following representation for the coefficient d_1^{N+1}

$$d_1^{N+1}(\lambda,\tau)=-\hat{p}_{N+1}(\lambda,\tau)+p^0_{N+1}(\lambda),$$

where $p^0_{N+1}(\lambda)$ is unknown constant (in t). With the help of analogous construction for the function $\mathbf{F}_{N+1}(x,\lambda,\tau)$ as $x\to+\infty$ we can receive representation for the coefficient d_2^{N+1}. The unknown constants can be found from initial data of Cauchy problem

$$\int_{-\infty}^{\infty}d\lambda\mathbf{F}_{N+1}(x,\lambda,0)=0.$$

Thus, we have

$$\begin{pmatrix}q^0_{N+1}(\lambda)\\ p^0_{N+1}(\lambda)\end{pmatrix}=F_0^{-1}(0)\left[i\int_0^x dyK_\lambda(x,y,0)\frac{\partial\mathbf{F}_N(y,\lambda,\tau)}{\partial\tau}|_{\tau=0}+\right.$$

$$+\hat{p}_{N+1}(\lambda,0)\mathbf{f}(x,\lambda,0)+\hat{q}_{N+1}(\lambda,0)\mathbf{g}(x,\lambda,0)].$$

Here $F_0^{-1}(0)$ is transformation which is determined by formula (9) for $\tau = 0$. Thus, we have single function $\mathbf{F}_{N+1}(x,\lambda,\tau)$ with properties (12) and (13). The theorem is proved.

4 Estimates of the formal solution

Consider part of the series (11)

$$\mathbf{\Phi}_N(x,t) = \int_{-\infty}^{+\infty} d\lambda \exp(i\lambda t)\sum_{l=0}^{N}\varepsilon^l \mathbf{F}_l(x,\lambda,\tau)$$

and corresponding discrepancy at the equation (10)

$$\Delta\mathbf{\Phi}_N(x,t) = -i\frac{\partial \mathbf{\Phi}_N}{\partial t} - \mathrm{H}(\tau)\mathbf{\Phi}_N =$$

$$= -i\varepsilon^{N+1}\int_{-\infty}^{\infty} d\lambda \exp(i\lambda t)\frac{\partial \mathbf{F}_N(y,\lambda,\tau)}{\partial \tau}. \qquad (18)$$

Theorem 2. *Discrepancy $\Delta\mathbf{\Phi}_N(x,t)$ satisfies following estimates*

$$\Delta\mathbf{\Phi}_N(x,t) \in C^\infty((-\infty,+\infty)\times[0,\tau_0]),$$

$$\forall k,l,m \quad \|\Delta\mathbf{\Phi}\|_{lkm} = O(\varepsilon^{N+1}).$$

Here

$$\|\mathbf{F}(x,t)\|_{lkm} = \sum_{i=1,2}\sum_{l'=0}^{l}\sum_{k'=0}^{k}\sum_{m'=0}^{m}\max_{x,\tau}\left|\frac{\partial^{l'+k'}F_i}{\partial^{l'}x\partial^{k'}t}(|x|^m+1)\right|.$$

The proof of this theorem is corollary of construction of formal solution $\hat{\mathbf{\Phi}}(x,t)$.

Theorem 3. *Formal series $\hat{\mathbf{\Phi}}$ is asymptotic expansion of precise solution $\mathbf{\Phi}$ of the Cauchy problem for $\tau \in [0,\tau_0]$. It satisfies following estimates*

$$\forall k,l,m \quad \|\mathbf{\Phi}-\mathbf{\Phi}_N\|_{lkm} = O(\varepsilon^{N+1})$$

Let introduce precise solution of Cauchy problem (10)

$$-i\mathbf{\Phi}_t = \mathrm{H}(\tau)\mathbf{\Phi}, \ \ \tau = \varepsilon t, \ \ \varepsilon \ll 1$$

$$\mathbf{\Phi}|_{t=o} = \phi_0(x).$$

This solution we can consider as result of action operator $M(t)$ on initial data of Cauchy problem

$$\mathbf{\Phi}(t) = M(t)\phi_0.$$

Operator H(τ) is selfadjoint in $L_2(-\infty,\infty)$. Thus $M(t)$ is unitary operator in $L_2(-\infty,\infty)$. Let us consider function

$$\mathbf{X}_N = \mathbf{\Phi} - \mathbf{\Phi}_N.$$

It satisfies nonhomogeneous equation

$$-i\frac{\partial \mathbf{X}_N}{\partial t} - H(\tau)\mathbf{X}_N = \Delta\mathbf{\Phi}_N. \tag{19}$$

We can calculate solution of this equation with the help of operator $M(t)$

$$\mathbf{X}_N = iM(t)\int_0^t M^{-1}(t')\Delta\mathbf{\Phi}_N(t')dt'.$$

From this relation we obtain estimate for the norm of function X_N

$$||\mathbf{X}_N(t)||_{L_2(-\infty,\infty)} \leq t \max_{t'\in[0,t]} ||\Delta\mathbf{\Phi}_N(t')||_{L_2(-\infty,\infty)}.$$

In order to obtain estimates for derivative $\frac{\partial \mathbf{X}_N}{\partial x}$ let us differentiate equation (19)

$$-i\frac{\partial^2 \mathbf{X}_N}{\partial t \partial x} - H(\tau)\frac{\partial \mathbf{X}_N}{\partial x} = \frac{\partial(\Delta\mathbf{\Phi}_N)}{\partial x} + \begin{pmatrix} 0 & p_x(x) \\ \overline{p_x(x)} & 0 \end{pmatrix}\mathbf{X}_N.$$

We again receive nonhomogeneous equation for the $\frac{\partial \mathbf{X}_N}{\partial x}$ that is analogous to (19). By analogous methods we can obtain estimates for $\frac{\partial \mathbf{X}_N}{\partial t}$ and $x^l\mathbf{X}_N$. The theorem is proved.

5 Large time asymptotic behaviour of the solution

Main term of our solution as $\varepsilon \to 0$ has a form

$$\mathbf{\Phi}_0(x,t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\lambda \exp(i\lambda t)(\mathbf{f}(x,\lambda,\tau),\mathbf{g}(x,\lambda,\tau))\times$$
$$\int_{-\infty}^{+\infty} dy \begin{pmatrix} \overline{\mathbf{f}^\top(y,\lambda,0)} \\ \overline{\mathbf{g}^\top(y,\lambda,0)} \end{pmatrix} \phi_0(y).$$

It is easy to see, that first integral has large parameter t. We can use corresponding asymptotic methods for calculation of this integral. As $t \gg 1$ integral is small if x is limited. As $x \to \pm\infty$ we can use asymptotics of the solutions $\mathbf{f}(x,\lambda,\tau)$ and $\mathbf{g}(x,\lambda,\tau)$ (5)-(8). Only solutions that have exponential $\exp(i\lambda t)\exp(-i\lambda x)$ as $x \to \infty$ or exponential $\exp(i\lambda t)\exp(i\lambda x)$ as $x \to -\infty$ give the contributions to asymptotics of the integral. Finally, we receive

$$\mathbf{\Phi}_0(x,t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\lambda \begin{pmatrix} \mathrm{e}^{i\lambda t - i\lambda x} & 0 \\ 0 & \mathrm{e}^{\mathrm{i}\lambda\mathrm{t}+\mathrm{i}\lambda\mathrm{x}} \end{pmatrix}$$

$$\int_{-\infty}^{+\infty} dy \left(\begin{array}{c} \overline{\mathbf{f}^{\top}(y,\lambda,0)} \\ \mathbf{g}^{\top}(y,\lambda,0) \end{array} \right) \phi_0(y) + \mathbf{o}(1).$$

In particular we see that solution is completely determined by operator H(0). This result is in accordance with results that can be received with the help of the adiabatic theorem in scattering theory.

The author would like to thanks V.S.Buslaev for his interest in the work, and A.A.Fedotov for his valuable remarks.

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