The effect of wind speedup in the formation of transverse dune fields

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Abstract

A computer simulation model for transverse-dune-field dynamics, corresponding to a uni-directional wind regime, is developed. In a previous formulation, two distinct problems were found regarding the cross-sectional dune shape, namely the erosion in the lee of dunes and the steepness of the windward slopes. The first problem is solved by introducing no erosion in shadow zones. The second issue is overcome by introducing a wind speedup factor, which can be accounted for by adding a term to the original transport length, which is proportional to the surface height. By incorporating these features we are able to model dunes whose individual shape and collective patterns are similar to those observed in nature. Moreover we show how introducing a non-linear wind speedup term leads to the reduction of dune height, and this may result in an equilibrium dune field configuration. This is thought to be because the non-linear wind speedup makes the sand trap efficiency lower than unity, even for higher dunes, so that the incoming and the outgoing sand flux are in balance. Finally we discuss the formation of the inter-dune flat surface arising from the existence of shadow zones.

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1 Introduction

Mathematical models are increasingly popular for the understanding of natural phenomena, and are employed to predict future states, possibly including human influences. Here we are interested in the morphology, dune growth, downwind migration and spatial patterns which form in a sand field. In order to develop a useful and accurate model some important factors need to be considered: the wind pattern over a dune must be correctly determined; the induced sand transport must be precisely estimated; and many other physical factors incorporated. Some progress has already been made especially with isolated barchan dunes as in the works of Howard *et al.* (1978), Wippermann and Gross (1985), Zeman and Jensen (1988), Weng *et al.* (1991). Though the question of equilibrium shape was not solved by Wippermann and Gross, they succeeded in reproducing a barchan dune with a good geometrical agreement to natural dunes. However extending this approach to a dune field, where many dunes grow, migrate and interact, is not an easy task. It is, for example, extremely difficult to calculate three dimensional wind patterns over many different forms of transient dune, such as partially connected dunes.

Recently, more abstract models to describe dune-field evolution have been developed by Werner (1995) and Nishimori and co-workers (Nishimori and Ouchi 1993, Nishimori *et al.* 1998). Though their mathematical formulations are not the same, they both assume that a dune field can be considered to be an accumulation of sand 'slabs' piled in a two-dimensional lattice, and that the movement of these slabs can be used to simulate wind directional sand transport and deformational shaping by gravity. Both these models succeeded in reproducing some types of dunes such as barchan dunes, transverse dunes, and linear dunes. They were also able to estimate the relations between dune shapes, wind directional variability, and sand availability, which are roughly confirmed with observations (Wasson and Hyde 1983, Livingstone and Warren 1996, figure 5.22 on p80).

Although there have been obvious successes, problems remain in these models. The most acute problem in Nishimori's model is that deformational shaping by gravity and directional sand transport occur at the same time scale, though in nature avalanches occur at faster time scales than bulk sand transport. In the Werner model the cross-sectional views of dunes are much more symmetric than in real dunes. Both models can nonetheless create dune field patterns, which approximately resemble natural ones.

We shall restrict our studies to transverse dunes, appearing in infinitely deep sand fields under a uni-directional wind regime. Transverse dunes are thought to be the simplest dunes as to shape and configuration. In their paper, Momiji and Warren emphasise wind speedup over a dune and sand trap efficiency as important factors when discussing equilibrium migration of transverse dunes (Momiji and Warren 1999).

In this paper, we analyse Werner's dune-field simulation model for the uni-directional wind regime leading to the evolution of transverse dunes. Following this, we introduce two revisions to the Werner model, that lead to dunes which more closely resemble those in nature. The first revision incorporates wind structure over a dune, and the second considers the wind speedup and the equilibrium dune shape.

2 Model and algorithm

Though some corrections and revisions are introduced in later sections, the basic model and algorithm used here are predominantly the same as used by Werner (1995). Dunes are built from sand slabs piled either on the 1-dimensional or on the 2-dimensional square lattice, whose edges are connected with periodic boundary conditions. The number of sand slabs is proportional to the surface height, and their movement comprises one-directional transport together with avalanching dynamics.

A sand slab is individually and randomly chosen for transport (erosion) from among all the slabs on the surface. The slab is moved a specific number of lattice sites, thus introducing a transport length, L, and is deposited at a site with a probability Ps (following Werner's notation). If the slab is not deposited, then it is repeatedly moved over L sites until deposition. Subsequently another slab is chosen randomly for transport. Shadow zones are introduced in the lee of dunes. If a slab is transported to a site in a shadow zone, it is deposited with unit probability. This process is repeated to construct the time evolution of the dune-field surface.

A maximum difference in surface elevation between adjacent lattice sites is enforced by introducing an angle of repose set to 33.7°. If the deposition of a sand slab at a particular site violates the angle-of-repose criterion, the slab is moved down the steepest gradient until compliance is achieved. Similarly, if erosion of a sand slab oversteepens the slope, starting at that lattice site, neighbouring sand slabs are moved downslope one lattice site, successively going up the steepest gradient until no angle exceeds the angle of repose. The only slab movement not in the transport direction originates from this form of avalanching.

The initial morphology is generated by placing sand slabs, one by one, on the lattice at random locations. In Werner's original paper, transverse dunes were simulated from the initial random morphology with 15 slabs per site on average. Here, since they appear in infinitely deep sand field, 100 slabs are piled on each site on average. Alternatively, dunes are constructed on the level surface with a depth of 100 slabs, as initial conditions. The windward slope angle of the initial dune was set to 10°. In the simulation time denoted by t is the number of lattice sites that have been polled for slab erosion divided by the number of surface lattice sites in the simulation. In all simulations, even after $t = 1,000,000$, there were no sites without a sand slab.

The parameters used in the simulations are given in table 1, which are almost the same as those used by Werner.

3 Dune shape and dynamics simulated with Werner's original model

Figure 1(a) shows transverse dunes on a 2-dimensional lattice simulated after $t = 500$, from a random initial morphology without any distinctive pattern or orientation, whose minimum and maximum number of slabs are 95 and 104, respectively. The windward slope angles are almost the same as the angle of repose of 33.7°, which is much larger than those of real dunes (about 10°). Consequently the resulting dunes have symmetrical cross sections, though as seen in figure 1(b) transverse dunes in nature have asymmetrical cross sections with the gentler windward slopes. As seen in figure 3A in Werner's paper, transverse dunes also have symmetrical cross sections.

Using the same model, figure 2 shows the cross-sectional views of an evolving isolated transverse dune on a level surface on a 2-dimensional lattice, whose initial number of slabs above the surface, h was 30. The simulation times, t are 0 to 100. The dune migrates downwind, during which its windward slope steepens and its slip face in the lee becomes shallower. However at the same time, the previously level surface behind and in front of the dune deform, and the windward slope destabilises. Meanwhile, behind the initial dune smaller accumulations form, which themselves grow as time evolves (e.g. close to $i = 625$). Lower dunes migrate more quickly, but are ephemeral, and as time passes, dune patterns of longer wavelength consisting of larger dunes grow and dominate the dune field. This wavelength selection is caused by avalanching.

Dune envelopes are shaped by sand accumulation and avalanching which suppresses short-wavelength undulations. With regard to avalanching first, it can be shown that the wavelength limits the amplitude of undulation, so that only longer-wavelength undulations can grow higher, resulting in dunes with a distinct shape. Assuming that an undulation, f has a wavelength of Λ and a maximum height of η_Λ, we may approximate the field as a sinusoidal function:

$$f = (\eta_\Lambda - \eta_{avg})(2\pi/\Lambda)\, sin((2\pi/\Lambda)x),$$

where x is a distance measure and η_{avg} is the average height. The local gradient, $\partial f/\partial x$ is

$$\partial f/\partial x = (\eta_\Lambda - \eta_{avg})(2\pi/\Lambda)\, cos((2\pi/\Lambda)x).$$

Since the gradient, $\partial f/\partial x$ cannot exceed 2/3 which corresponds to the angle of repose of 33.7° then

$$\eta_\Lambda - \eta_{avg} \leq (\Lambda/2\pi)\, tan 33.7^\circ = (\Lambda/2\pi)(2/3) \simeq 0.11\Lambda.$$

Consequently, shorter-wavelength undulations can only achieve lower heights, and thus only longer-wavelength undulations can grow higher.

Next, it can be shown that sand accumulation is due to the existence of the shadow zones in the lee of dunes. Without a shadow zone, the sand slab conservation equation can be described as

$$\begin{aligned} h_{n+1}(i,j) &= h_n(i,j) + (\text{transported-incoming } \underline{\text{s}}\text{and } \underline{\text{s}}\text{labs}) \\ &- (\text{transported-outgoing (erosion) s. s.}) + \text{avalanche} \\ &= h_n(i,j) + (\sum_{i'=i-kL} (1-Ps)^{k-L} Ps + \text{by-passing s. s.}) \\ &- (1 + \text{by-passing s. s.}) + \text{avalanche}, \qquad (1) \end{aligned}$$

where $h_n(i,j)$ is the number of slabs at the site (i,j) at time n. Given that the system is infinitely large, the second term of the right-hand side of (1) is

$$\sum_{i'=i-kL} (1-Ps)^{k-L} Ps \rightarrow Ps \sum_{k}^{\infty} (1-Ps)^{k-1} = Ps/\{1-(1-Ps)\} = 1.$$

Since avalanching occurs equally at each site, equation (1) implies that

$$h_{n+1}(i,j) - h_n(i,j) = 0 \qquad (2)$$

on average taken over all sites with a periodic boundary condition. In the case of infinite sand depth, producing transverse dunes in Werner's model, dunes grow because incoming sand is accumulated once it has been deposited in the shadow zone, either directly or via avalanching. Without a shadow zone, from equations (1) and (2) the sand flux is found to be constant at every site.

4 No erosion in shadow zones

In Werner's original model the surface in the lee of a dune is eroded. In his model, shadow zones are applied only to sand slab deposition, and erosion still occurs in the shadow zones. However, field measurements show that wind speeds within the separation cell, which corresponds to the shadow zone here, are typically insufficient to cause sand movement (Frank and Kocurek 1996). Figure 3 shows cross-sectional views of transverse dunes simulated without erosion in shadow zones, on a 2-dimensional lattice from an initially isolated transverse dune with number of slabs, h of 30, built on a level surface. The simulation times, t are 0 to 100. With no erosion in the shadow zones: a) the slip face in the lee is sustained; b) the dunes grow more quickly due to strong accumulation of sand slabs; and c) the dunes migrate downwind with constant speed. These features are similar to those of dunes in nature. Figure 4 shows transverse dunes simulated on a 2-dimensional lattice from an initially random morphology. Without erosion in the shadow zones, the dunes are higher than those shown in figure 1, due to strong slab accumulation. In the remainder of this paper we consider no erosion in shadow zones in all simulations.

5 Introducing wind speedup

The wind speedup over a windward slope of a dune is necessary to sustain the shape of the windward slope (Bagnold 1941, Zeman and Jensen 1988, Momiji and Warren 1999). This wind speedup has been noted, both in the field and in experiments (see for example Lancaster *et al.* 1996, Mulligan 1988 and Wiggs *et al.* 1996). Lancaster and co-workers found that the wind speed increases linearly over a barchan dune, provided that the wind is strong enough such that the shear velocity significantly exceeds the threshold shear velocity. A similar wind speedup on a transverse dune was observed by Mulligan.

In his revised model, Nishimori and co-workers attempted to introduce a more realistic slab transport rule compared to Werner's approach, based on the 1-dimensional wind flow calculation (2-dimensional wind flow pattern) over a well-developed dune, in which wind speedup is included (Nishimori *et al.* 1998).

In this section, we attempt to introduce a wind speedup effect into Werner's model with a kinematic rule. Following this, we examine the effect of wind speedup on a dune shape with some simulations.

5.1 Kinematic formulation

If dunes migrate downwind at the constant speed, c_d without changing their shapes as is widely believed (Bagnold 1941), then the local height, $\eta(x,t)$ follows the advection equation:

$$\partial\eta/\partial t + c_d\,\partial\eta/\partial x = 0, \tag{3}$$

Combining (3) and the sand conservation equation:

$$\gamma\,\partial\eta/\partial t + \partial q/\partial x = 0,$$

where γ is the sand bulk density in dunes and q is sand flux, we obtain

$$\gamma\, c_d\,\partial\eta/\partial x = \partial q/\partial x. \tag{4}$$

Equation (4) must always be satisfied when we discuss the migration of shape-invariant dunes (Zeman and Jensen 1988). Integrating (4) from $x = -\infty$ to $x = 0$ corresponding to the dune crest leads to

$$q(x) - q(-\infty) = \gamma\, c_d\, \eta(x). \tag{5}$$

Let us assume that sand flux, $q(x)$ obeys Bagnold's sand transport formula:

$$q(x) = C(d/D)^{1/2}(\rho_a/g)(u_*(x))^3,$$

where C is a constant, d is sand grain diameter, D is reference diameter of 0.25 mm, ρ_a is air density, g is gravitational acceleration and $u_*(x)$ is wind shear velocity. Assuming that the shear velocity on a dune surface is $\delta u_*(x)$ larger than that on a bare ground, due to the wind speedup over the dune, the corresponding increase of sand flux is written as

$$\begin{aligned} q(x) - q(-\infty) &= C(d/D)^{1/2}(\rho_a/g)[u_*(x)^3 - u_*(-\infty)^3] \\ &= C(d/D)^{1/2}(\rho_a/g)[(u_*(-\infty) + \delta u_*(x))^3 - u_*(-\infty)^3] \\ &= C(d/D)^{1/2}(\rho_a/g)u_*(-\infty)^3[\{1 + (\delta u_*(x)/u_*(-\infty))\}^3 - 1]. \end{aligned} \tag{6}$$

If wind speedup is small enough:

$$\delta u_*(x) << u_*(-\infty),$$

equation (6) becomes

$$\begin{aligned} q(x) - q(-\infty) &= C(d/D)^{1/2}(\rho_a/g)u_*(-\infty)^3[\{1 + 3(\delta u_*(x)/u_*(-\infty))\} - 1] \\ &\approx 3C(d/D)^{1/2}(\rho_a/g)u_*(-\infty)^2\delta u_*(x). \end{aligned} \tag{7}$$

Combining (5) and (7)

$$\delta u_*(x) \approx [(\gamma\, c_d)/3C(d/D)^{1/2}(\rho_a/g)u_*(-\infty)^2]\eta(x). \tag{8}$$

Though in Werner's model, both the transport length of sand slabs, L and the number of slabs to be removed, δh, are constant (typically $L = 5$ and $\delta h = 1$), L is assumed here to be proportional to $u_*(x)$, so that from equation (8),

$$L(x) = L_0 + \delta Lc\, h(x), \tag{9}$$

where $h(x)$ is the number of slabs at x. Given that the sand flux, $q(x)$ can be approximated as

$$q(x) = L(x)\, \delta h(x)/3$$

where the coefficient, 3 is the inverse slab aspect ratio (Nishimori *et al.* 1998), since

$$q(-\infty) = L_0\, \delta h(x)/3 \approx u_*(-\infty)^3,$$

and

$$\delta q(x) = \delta L(x)\, \delta h(x)/3 \approx u_*(-\infty)^2 \delta u_*(x),$$

$\delta h(x)$ must be given by

$$\delta h(x) = u_*(-\infty)^2 = const. \tag{10}$$

Consequently from equations (9) and (10), the wind speedup over a dune can be introduced by letting the transport length of sand slab, L be

$$L(x) = L_0 + \delta Lc\,(h(x) - h_{avg}), \tag{11}$$

where x is the erosion/ongoing 'bounce' site and h_{avg} is the average number of slabs, here taken as 100. Equation (11) can be interpreted as the linear wind speedup observed by Lancaster *et al.* (1996) and Mulligan (1988).

5.2 Simulations

Simulations were carried out with and without a wind speedup term introduced in the form of equation (11). Figure 5 shows the simulation results started from the same initial transverse dune used in section 4, with various wind speedup coefficients, δLc. Introducing wind speedup makes: a) dunes lower; b) dunes migrate more quickly; and c) dune windward slopes gentler, producing asymmetric dunes. For $\delta Lc = 0.4$, windward slopes with the angle of 10° are sustained. Figure 6 shows asymmetric transverse dunes formed on a 2-dimensional lattice from a random initial morphology by introducing wind speedup. The simulation time, t is 1,500, and the wind speedup coefficient, δLc is 0.4. The resulting transverse dunes are the most qualitatively similar to natural ones achieved yet.

The time evolution of a 1-dimensional dune field with the wind speedup coefficient, δLc of 0.4 is shown in figure 7. The wavelength of the transverse dunes increases as time passes. For run times longer than that displayed in the figure, the equilibrium dune field seems to converge to only an isolated, asymmetric dune, which is surrounded by completely plane, bare ground. Consequently, the dune-to-dune spacing is equal to the lattice size, 1,024, meaning that the model is not adequate when the dune-to-dune spacing approaches the lattice size.

The reason for this equilibrium dune field configuration is thought to be related to sand trap efficiency, TE, which is the proportion of moving sand trapped in the dune slip face in the lee (Cooke *et al.* 1992). At equilibrium, sand flux coming to the dune must be the same as the outgoing flux (Zeman and Jensen 1988, Momiji and Warren 1999). However, since the shadow zone extends more quickly than the slab transport length at the crest of dune, L increases as the dune grows higher, all the slabs crossing the dune crest will be captured in the dune, meaning the sand trap efficiency, $TE = 1.0$, so that nothing regulates dune height. Wind speedup introduced with the form of equation (11), seems to regulate only the windward slope angle.

6 Non-linear wind speedup and equilibrium dune field

From their microscale analysis of saltating sand grains, Anderson and Hallet established a set of equations describing saltation length, λ against shear velocity, u_* (Anderson and Hallet 1986, Anderson 1988). Figure 8 shows the relation between the saltation length, λ and the shear velocity, u_*, calculated using their equations (2) to (5b) in the 1988 paper. Other parameters used here are sand grain diameter of 0.25 mm and lift-off angle of 50° (White and Schulz 1977). Considering this non-linear relation, it seems to be reasonable that the non-linear wind speedup term, $\delta Lc2$ is introduced into slab transport length, L in our model, so that equation (11) can be revised as

$$L(x) = L_0 + \delta Lc\,(h(x) - h_{avg}) + \delta Lc2\,(h(x) - h_{avg})^2. \tag{12}$$

Figure 9 shows the time evolution of 1-dimensional dune field with non-linear wind speedup introduced in the form of equation (12). The wind speedup coefficient, δLc is 0.4 and the non-linear wind speedup coefficient, $\delta Lc2$ is taken as 0.002. Comparing figures 7 and 9, we find that introducing non-linear wind speedup a) makes dunes lower and b) increases the number of dunes. Figure 10 shows the relations between maximum dune height and simulation time, t in the cases with and without non-linear wind speedup. δLc and $\delta Lc2$ is are the same as in figure 9. In the case with non-linear wind speedup, the maximum dune height increases more slowly. The dune height far after $t = 1,000,000$ is thought to be capped by the non-linear wind speedup effect, since some part of slab flux crossing at the crest of a dune goes outside the shadow zone, reducing sand trap efficiency, TE below 1.0, so that the incoming and outgoing slab fluxes balance. To check if the dune height saturates, or not, more simulations are needed with longer time, t. Figure 11 shows transverse dunes simulated on the 2-dimensional lattice, from a random initial morphology. δLc and $\delta Lc2$ are 0.4 and 0.002, respectively, as in figures 9 and 10. Simulation time, t is 1,500. Some dunes in figure 11 are lower than corresponding dunes in figure 6 because of the non-linear wind speedup. Considering figure 10, this effect is thought to appear clearly if the simulation time, t is much longer.

7 Formation of inter-dune flat surfaces

As seen in figures 7 and 9, after long enough time, flat surfaces appear between dunes. These inter-dune flat surfaces are thought to be related to the shadow zones in the lee of dunes. Following Fredsøe (1982), we consider the relationship between the local sand flux, $q(x)$, the local height, $\eta(x)$, the sand flux at the crest, q_{cr}, and the dune height, H. If dunes migrate downwind at a constant speed, c_d without changing their shapes, as assumed in previous sections, the dune migration speed, c_d can be written as

$$c_d = q_{cr}\,TE/\gamma\,H \tag{13}$$

(see for example Momiji and Warren 1999). Since we are considering the dunes in equilibrium state, incoming and outgoing sand flux are the same:

$$q(-\infty) = q(+\infty) = (1 - TE)\,q_{cr}. \tag{14}$$

Combining (4), (13) and (14), we get

$$(q(x) - q(-\infty))/(q_{cr} - q(-\infty)) = \eta(x)/H. \tag{15}$$

A simpler version of equation (15) was derived by Fredsøe (1982) discounting the sand trap efficiency, TE.

From equation (15) we know that if the local height, $\eta(x)$ is constant, there is no change in sand flux, $q(x)$. Since there is no erosion in the shadow zone, change in sand flux, $q(x)$ within the shadow zone can occur only by deposition. However, most of the sand slabs crossing the crest are captured in the slip face in the lee of the dune. The flat surface in the lee of the dune implies that most of the rest of slabs not captured in the slip face become by-passing slabs, which corresponds to $q(+\infty)$ in equation (14).

Some transverse dune systems observed in nature do not have distinct inter-dune flat surfaces. To describe those features more precise dynamics in the lee of the dune must be incorporated.

8 Conclusions

A simulation model for a transverse-dune field, corresponding to the uni-directional wind regime, is developed based on Werner's model (1995). In the present work, two distinct problems were overcome. Inconsistencies with the cross-sectional dune shape can be solved by introducing no erosion in shadow zones. While the steepness of the windward slope can be correctly modelled introducing wind speedup over a dune, which can be formulated by adding the height-proportional increase term to the original transport length, L. Introducing a non-linear wind speedup term can reduce the dune height, which may result in an equilibrium dune field configuration. This is thought to be because the non-linear wind speedup makes the sand trap efficiency, TE lower than 1.0, even for high dunes, so that the incoming and the outgoing sand fluxes balance. The formation of inter-dune flat surfaces is thought to be related to the shadow zone.

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Table 1

parameter	value
lattice size:	
2-d lattice	1,024(1,024*
1-d lattice*	1,024*
slab aspect ratio	1/3
angle of repose	$\tan^{-1}(2/3)=33.7°$
shadow zone angle	15°
transport length	$L = 5$**
erosion probability	same at every site**
deposition probability:	
at a site outside shadow zone	$Ps = 0.6$
at a site in shadow zones	$Ps = 1.0$
average number of slabs at a site:	100*

* different from Werner's
** revised in latter section

Figure captions

Figure 1 Transverse dunes (a) in (i, j) space simulated with Werner's original model, from a random initial morphology, and (b) in real nature. The simulation time, t is 500. Contour lines are for 100, 105 and 110 slabs.

Figure 2 Evolution of an initially introduced, isolated transverse dune, simulated with Werner's original model. The dune's initial number of slabs above the surface shown by a dotted line is 30. The total simulation time, t is 100.

Figure 3 Evolution of an initially introduced, isolated transverse dune simulated without erosion in shadow zones. The dune's initial number of slabs above the surface is 30. The simulation time, t is 100.

Figure 4 Transverse dunes simulated from an initially random morphology. The simulation time, t is 500. No erosion occurs in shadow zones. Contour lines are for 100, 110, 120, 130 and 140 slabs.

Figure 5 Evolution of an initially introduced, isolated transverse dune simulated with various wind speedup coefficients, δLc. The dune's initial number of slabs above the surface is 30. The simulation time, t is 100.

Figure 6 Asymmetric transverse dunes formed from a random initial morphology by introducing wind speedup. The simulation time, t is 1,500, and the wind speedup coefficient, δLc is 0.4. Contour lines are for 100, 105, 110 and 115 slabs.

Figure 7 Time evolution of a 1-dimensional dune field, from a random initial morphology. The wind speedup coefficient, δLc is 0.4..

Figure 8 Relation between saltation length, λ and shear velocity, u_*. Data are plotted following Anderson (1988). Other parameters, not specified in his paper but used here, are sand grain diameter of 0.25 mm and lift-off angle of 50° (White and Schulz 1977).

Figure 9 Time evolution of transverse dunes simulated on the 1-dimensional lattice with non-linear wind speedup, from a random initial morphology. Linear wind speedup coefficient, δLc is 0.4. Non-linear wind speedup coefficient, $\delta Lc2$ is 0.002.

Figure 10 Relations between maximum dune height and simulation time, t. The relations with and without the non-linear wind speedup are averaged from 8 and 15 runs on the 1-dimensional lattice, respectively. Linear wind speedup coefficient, δLc is 0.4. Non-linear wind speedup coefficient, $\delta Lc2$ is 0.002.

Figure 11 Transverse dunes simulated on the 2-dimensional lattice, from a random initial morphology. Linear wind speedup coefficient, δLc is 0.4. Non-linear wind speedup coefficient, $\delta Lc2$ is 0.002. The simulation time, t is 1,500. Contour lines are for 100, 105 and 110 slabs.

Corrections

A few corrections were made after the submission. None of these changes the original context.

Original eq. (6):

$$\begin{aligned} q(x) - q(-\infty) &= C(d/D)^{1/2}(\rho_a/g)[u_*(x)^3 - u_*(-\infty)^3] \\ &= C(d/D)^{1/2}(\rho_a/g)[(u_*(-\infty) + \delta u_*(x))^3 - u_*(-\infty)^3] \\ &= C(d/D)^{1/2}(\rho_a/g)u_*(-\infty)[\{1 + (\delta u_*(x)/u_*(-\infty))\}^3 - 1]. \end{aligned}$$

Corrected eq. (6):

$$\begin{aligned} q(x) - q(-\infty) &= C(d/D)^{1/2}(\rho_a/g)[u_*(x)^3 - u_*(-\infty)^3] \\ &= C(d/D)^{1/2}(\rho_a/g)[(u_*(-\infty) + \delta u_*(x))^3 - u_*(-\infty)^3] \\ &= C(d/D)^{1/2}(\rho_a/g)u_*(-\infty)^3[\{1 + (\delta u_*(x)/u_*(-\infty))\}^3 - 1]. \end{aligned}$$

Original eq. (7):

$$\begin{aligned} q(x) - q(-\infty) &= C(d/D)^{1/2}(\rho_a/g)u_*(-\infty)[\{1 + 3(\delta u_*(x)/u_*(-\infty))\} - 1] \\ &\approx 3C(d/D)^{1/2}(\rho_a/g)\delta u_*(x). \end{aligned}$$

Corrected eq. (7):

$$\begin{aligned} q(x) - q(-\infty) &= C(d/D)^{1/2}(\rho_a/g)u_*(-\infty)^3[\{1 + 3(\delta u_*(x)/u_*(-\infty))\} - 1] \\ &\approx 3C(d/D)^{1/2}(\rho_a/g)u_*(-\infty)^2\delta u_*(x). \end{aligned}$$

Original eq. (8)

$$\delta u_*(x) \approx [(\gamma\, c_d)/3C(d/D)^{1/2}(\rho_a/g)]\eta(x).$$

Corrected eq. (8)

$$\delta u_*(x) \approx [(\gamma\, c_d)/3C(d/D)^{1/2}(\rho_a/g)u_*(-\infty)^2]\eta(x).$$

Original eq. of one before eq. (10):

$$\delta q(x) = \delta L(x)\, \delta h(x)/3 \approx \delta u_*(x),$$

Corrected eq.:

$$\delta q(x) = \delta L(x)\, \delta h(x)/3 \approx u_*(-\infty)^2\delta u_*(x),$$