A Note on Constructions of Quantum-Field Operators

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Abstract. We show some tune points in constructions of field operators in quantum field theory (QFT). They are related to the old discussions on interpretations of the negative-energy solutions of relativistic equations. It is easy to check that both algebraic equation $Det(\hat{p}-m)=0$ and $Det(\hat{p}+m)=0$ for $u-$ and $v-$ 4-spinors have solutions with $p_0 = \pm E_p = \pm\sqrt{\mathbf{p}^2+m^2}$. The same is true for higher-spin equations. Meanwhile, every book considers the equality $p_0 = E_p$ for both $u-$ and $v-$ spinors of the $(1/2,0)\oplus(0,1/2)$ representation only, thus applying the Dirac-Feynman-Stueckelberg procedure for elimination of the negative-energy solutions. The recent Ziino works (and, independently, the articles of several others) show that the Fock space can be doubled. We re-consider this possibility on the quantum field level for both $s=1/2$ and higher spin particles.

Keywor: QFT, Dark Matter, Dirac Equation.

1. Introduction

The recent problems of superluminal neutrinos, e. g., Ref. [1], negative-mass squared neutrinos, e. g. [2], various schemes of oscillations including sterile neutrinos, e. g. [3], require attention. The problem of the lepton mass splitting (e,μ,τ) has long history [4]. This suggests that something missed in the foundations of relativistic quantum theories. Modifications seem to be necessary in the Dirac sea concept, and in the even more sophisticated Stueckelberg concept of the backward propagation in time. The Dirac sea concept is intrinsically related to the Pauli principle. However, the Pauli principle is intrinsically connected with the Fermi statistics and the anticommutation relations of fermions. Recently, the concept of the *bi-orthonormality* has been proposed; the (anti) commutation relations and statistics are assumed to be different for *neutral* particles [5]. One can speculate that they go in the negative-energy sea, but due to some reasons (interaction?) they do not live there (from our viewpoint), but return back (been expelled), thus showing us the new kind of oscillations on the Planck scale $\omega \sim E/\hbar$, Ref. [6]. Perhaps, some of the neutrinos remain *sterile* even in our world.

We propose the relevant modifications in the basics of the relativistic quantum theory for *neutral* particles below. However much work is still needed.

2. The General Framework and Connections with Previous Models

The Dirac equation is:

$$[i\gamma^\mu \partial_\mu - m]\Psi(x) = 0\,. \tag{1}$$

At least, 3 methods of its derivation exist [7, 8, 9]:

- the Dirac one (the Hamiltonian should be linear in $\partial/\partial x^i$, and be compatible with $E_p^2 - \mathbf{p}^2 c^2 = m^2 c^4$);
- the Sakurai one (based on the equation $(E_p - \sigma\cdot\mathbf{p})(E_p + \sigma\cdot\mathbf{p})\phi = m^2\phi$);
- the Ryder one (the relation between 2-spinors at rest is $\phi_R(\mathbf{0}) = \pm\phi_L(\mathbf{0})$, and boosts).

The γ^μ are the Clifford algebra matrices

$$\gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2g^{\mu\nu}\,. \tag{2}$$

Usually, everybody uses the following definition of the field operator [10] in the pseudo-Euclidean metrics:

$$\Psi(x) = \frac{1}{(2\pi)^3}\sum_h \int \frac{d^3\mathbf{p}}{2E_p}[u_h(\mathbf{p})a_h(\mathbf{p})e^{-ip\cdot x} + v_h(\mathbf{p})b_h^\dagger(\mathbf{p})]e^{+ip\cdot x}]\,, \tag{3}$$

as given *ab initio*. After actions of the Dirac operator at $\exp(\mp i p_\mu x^\mu)$ the 4-spinors ($u-$ and $v-$) satisfy the momentum-space equations: $(\hat{p} - m)u_h(p) = 0$ and $(\hat{p} + m)v_h(p) = 0$, respectively; the h is the polarization index. It is easy to prove from the characteristic equations $Det(\hat{p} \mp m) = (p_0^2 - \mathbf{p}^2 - m^2)^2 = 0$ that the solutions should satisfy the energy-momentum relation $p_0 = \pm E_p = \pm\sqrt{\mathbf{p}^2 + m^2}$.

The general scheme of construction of the field operator has been presented in [11]. In the case of the $(1/2, 0) \oplus (0, 1/2)$ representation we have:

$$\begin{aligned}
&\Psi(x) = \frac{1}{(2\pi)^3}\int d^4p\,\delta(p^2 - m^2)e^{-ip\cdot x}\Psi(p) = \\
&= \frac{1}{(2\pi)^3}\sum_h \int d^4p\,\delta(p_0^2 - E_p^2)e^{-ip\cdot x}u_h(p_0,\mathbf{p})a_h(p_0,\mathbf{p}) = \\
&= \frac{1}{(2\pi)^3}\int \frac{d^4p}{2E_p}[\delta(p_0 - E_p) + \delta(p_0 + E_p)][\theta(p_0) + \theta(-p_0)]e^{-ip\cdot x} \\
&\quad \sum_h u_h(p)a_h(p) = \frac{1}{(2\pi)^3}\sum_h \int \frac{d^4p}{2E_p}[\delta(p_0 - E_p) + \delta(p_0 + E_p)] \\
&\quad \left[\theta(p_0)u_h(p)a_h(p)e^{-ip\cdot x} + \theta(p_0)u_h(-p)a_h(-p)e^{+ip\cdot x}\right] \\
&= \frac{1}{(2\pi)^3}\sum_h \int \frac{d^3\mathbf{p}}{2E_p}\theta(p_0)\left[u_h(p)a_h(p)|_{p_0=E_p}e^{-i(E_p t - \mathbf{p}\cdot\mathbf{x})} + \right. \\
&+ \left. u_h(-p)a_h(-p)|_{p_0=E_p}e^{+i(E_p t - \mathbf{p}\cdot\mathbf{x})}\right]\,.
\end{aligned} \tag{4}$$

During the calculations above we had to represent $1=\theta(p_0)+\theta(-p_0)$ in order to get positive- and negative-frequency parts.[1] Moreover, during these calculations we did not yet assumed, which equation this field operator (namely, the $u-$ spinor) satisfies, with negative- or positive- mass?

In general we should transform $u_h(-p)$ to the $v(p)$. The procedure is the following one [13]. In the Dirac case we should assume the following relation in the field operator:

$$\sum_h v_h(p) b_h^\dagger(p) = \sum_h u_h(-p) a_h(-p)\,. \tag{5}$$

We know that [9]

$$\begin{aligned}
\bar u_\mu(p) u_\lambda(p) &= +m\delta_{\mu\lambda}\,, & (6)\\
\bar u_\mu(p) u_\lambda(-p) &= 0\,, & (7)\\
\bar v_\mu(p) v_\lambda(p) &= -m\delta_{\mu\lambda}\,, & (8)\\
\bar v_\mu(p) u_\lambda(p) &= 0\,, & (9)
\end{aligned}$$

but we need $\Lambda_{\mu\lambda}(p)=\bar v_\mu(p) u_\lambda(-p)$. By direct calculations, we find

$$-m b_\mu^\dagger(p) = \sum_\lambda \Lambda_{\mu\lambda}(p) a_\lambda(-p)\,. \tag{10}$$

Hence, $\Lambda_{\mu\lambda}=-im(\boldsymbol{\sigma}\cdot\mathbf{n})_{\mu\lambda}$, $\mathbf{n}=\mathbf{p}/|\mathbf{p}|$, and

$$b_\mu^\dagger(p) = i\sum_\lambda (\boldsymbol{\sigma}\cdot\mathbf{n})_{\mu\lambda} a_\lambda(-p)\,. \tag{11}$$

Multiplying (5) by $\bar u_\mu(-p)$ we obtain

$$a_\mu(-p) = -i\sum_\lambda (\boldsymbol{\sigma}\cdot\mathbf{n})_{\mu\lambda} b_\lambda^\dagger(p)\,. \tag{12}$$

The equations are self-consistent. In the $(1,0)\oplus(0,1)$ representation the similar procedure leads to somewhat different situation:

$$a_\mu(p) = [1-2(\mathbf{S}\cdot\mathbf{n})^2]_{\mu\lambda} a_\lambda(-p)\,. \tag{13}$$

This signifies that in order to construct the Sankaranarayanan-Good field operator (which was used by Ahluwalia, Johnson and Goldman [Phys. Lett. B (1993)], it satisfies $[\gamma_{\mu\nu}\partial_\mu\partial_\nu - \frac{(i\partial/\partial t)}{E} m^2]\Psi(x)=0$, we need additional postulates. For instance, one can try to construct the left- and the right-hand side of the field operator separately each other [12].

However, other ways of thinking are possible. First of all to mention, we have, in fact, $u_h(E_p,\mathbf{p})$ and $u_h(-E_p,\mathbf{p})$ originally, which may satisfy the equations:[2]

$$\left[E_p(\pm\gamma^0) - \boldsymbol{\gamma}\cdot\mathbf{p} - m\right] u_h(\pm E_p,\mathbf{p}) = 0\,. \tag{14}$$

[1] See Ref. [12] for some discussion.

[2] Remember that, as before, we can always make the substitution $\mathbf{p}\to-\mathbf{p}$ in any of the integrands of (4).

Due to the properties $U^\dagger\gamma^0 U = -\gamma^0$, $U^\dagger\gamma^i U = +\gamma^i$ with the unitary matrix $U = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = \gamma^0\gamma^5$ in the Weyl basis,[3] we have

$$\left[E_p\gamma^0 - \boldsymbol{\gamma}\cdot\mathbf{p} - m\right] U^\dagger u_h(-E_p,\mathbf{p}) = 0\,. \tag{15}$$

Thus, unless the unitary transformations do not change the physical content, we have that the negative-energy spinors $\gamma^5\gamma^0 u^-$ (see (15)) satisfy the accustomed "positive-energy" Dirac equation. Their explicite forms $\gamma^5\gamma^0 u^-$ are different from the textbook "positive-energy" Dirac spinors. They are the following ones (the linear combination of the positive-energy spinors):[4]

$$\tilde{u}(p) = \frac{N}{\sqrt{2m(-E_p+m)}}\begin{pmatrix} -p^+ + m \\ -p_r \\ p^- - m \\ -p_r \end{pmatrix}\,, \tag{16}$$

$$\tilde{\tilde{u}}(p) = \frac{N}{\sqrt{2m(-E_p+m)}}\begin{pmatrix} -p_l \\ -p^- + m \\ -p_l \\ p^+ - m \end{pmatrix}\,. \tag{17}$$

$E_p = \sqrt{\mathbf{p}^2+m^2} > 0$, $p_0 = \pm E_p$, $p^\pm = E \pm p_z$, $p_{r,l} = p_x \pm i p_y$. Their normalization is to $(-2N^2)$.

What about the $\tilde{v}(p) = \gamma^0 u^-$ transformed with the γ_0 matrix? They are not equal to $v_h(p) = \gamma^5 u_h(p)$. Obviously, they also do not have well-known forms of the usual $v-$ spinors in the Weyl basis, differing by phase factor and in the sign at the mass term.

Next, one can prove that the matrix

$$P = e^{i\theta}\gamma^0 = e^{i\theta}\begin{pmatrix} 0 & 1_{2\times 2} \\ 1_{2\times 2} & 0 \end{pmatrix} \tag{18}$$

can be used in the parity operator as well as in the original Weyl basis. The parity-transformed function $\Psi'(t,-\mathbf{x}) = P\Psi(t,\mathbf{x})$ must satisfy

$$[i\gamma^\mu\partial'_\mu - m]\Psi'(t,-\mathbf{x}) = 0\,, \tag{19}$$

with $\partial'_\mu = (\partial/\partial t, -\nabla_i)$. This is possible when $P^{-1}\gamma^0 P = \gamma^0$ and $P^{-1}\gamma^i P = -\gamma^i$. The matrix (18) satisfies these requirements, as in the textbook case. However, if we would take the phase factor to be zero we obtain that while $u_h(p)$ have the eigenvalue $+1$, but $(R = (\mathbf{x}\to -\mathbf{x}, \mathbf{p}\to -\mathbf{p}))$

$$PR\tilde{u}(p) = PR\gamma^5\gamma^0 u(-E_p,\mathbf{p}) = -\tilde{u}(p)\,, \tag{20}$$

$$PR\tilde{\tilde{u}}(p) = PR\gamma^5\gamma^0 u(-E_p,\mathbf{p}) = -\tilde{\tilde{u}}(p)\,. \tag{21}$$

[3]The properties of the $U-$ matrix are opposite to those of $P^\dagger\gamma^0 P = +\gamma^0$, $P^\dagger\gamma^i P = -\gamma^i$ with the usual $P = \gamma^0$, thus giving $\left[-E_p\gamma^0 + \gamma\cdot\mathbf{p} - m\right] P u_h(-E_p,\mathbf{p}) = -\left[\hat{p}+m\right]\tilde{v}_?(E_p,\mathbf{p}) = 0$. While, the relations of the spinors $v_h(E_p,\mathbf{p}) = \gamma_5 u_h(E_p,\mathbf{p})$ are well-known, it seems that the relations of the $v-$ spinors of the positive energy to $u-$ spinors of the negative energy are frequently forgotten, $\tilde{v}_?(E_p,\mathbf{p}) = \gamma^0 u_h(-E_p,\mathbf{p})$.

[4]We use tildes because we do not yet know their polarization properties.

Perhaps, one should choose the phase factor $\theta = \pi$. Thus, we again confirmed that the relative (particle-antiparticle) intrinsic parity has physical significance only.

Similar formulations have been presented in Refs. [14], and [15]. The group-theoretical basis for such doubling has been given in the papers by Gelfand, Tsetlin and Sokolik [16], who first presented the theory in the 2-dimensional representation of the inversion group in 1956 (later called as "the Bargmann-Wightman-Wigner-type quantum field theoryïn 1993).

M. Markov wrote long ago *two* Dirac equations with opposite signs at the mass term [14].

$$
\begin{aligned}
[i\gamma^\mu \partial_\mu - m]\,\Psi_1(x) &= 0\,, && (22)\\
[i\gamma^\mu \partial_\mu + m]\,\Psi_2(x) &= 0\,. && (23)
\end{aligned}
$$

In fact, he studied all properties of this relativistic quantum model (while he did not know yet the quantum field theory in 1937). Next, he added and subtracted these equations. One has

$$
\begin{aligned}
i\gamma^\mu \partial_\mu \varphi(x) - m\chi(x) &= 0\,, && (24)\\
i\gamma^\mu \partial_\mu \chi(x) - m\varphi(x) &= 0\,. && (25)
\end{aligned}
$$

Thus, φ and χ solutions can be presented as some superpositions of the Dirac 4-spinors $u-$ and $v-$. These equations, of course, can be identified with the equations for the Majorana-like $\lambda-$ and $\rho-$, which we presented in Ref. [17].[5]

$$
\begin{aligned}
i\gamma^\mu \partial_\mu \lambda^S(x) - m\rho^A(x) &= 0\,, && (26)\\
i\gamma^\mu \partial_\mu \rho^A(x) - m\lambda^S(x) &= 0\,, && (27)\\
i\gamma^\mu \partial_\mu \lambda^A(x) + m\rho^S(x) &= 0\,, && (28)\\
i\gamma^\mu \partial_\mu \rho^S(x) + m\lambda^A(x) &= 0\,. && (29)
\end{aligned}
$$

Neither of them can be regarded as the Dirac equation. However, they can be written in the 8-component form as follows:

$$
\begin{aligned}
[i\Gamma^\mu \partial_\mu - m]\,\Psi_{(+)}(x) &= 0\,, && (30)\\
[i\Gamma^\mu \partial_\mu + m]\,\Psi_{(-)}(x) &= 0\,, && (31)
\end{aligned}
$$

with

$$
\Psi_{(+)}(x) = \begin{pmatrix} \rho^A(x) \\ \lambda^S(x) \end{pmatrix}, \Psi_{(-)}(x) = \begin{pmatrix} \rho^S(x) \\ \lambda^A(x) \end{pmatrix}, \qquad (32)
$$

$$
\text{and} \quad \Gamma^\mu = \begin{pmatrix} 0 & \gamma^\mu \\ \gamma^\mu & 0 \end{pmatrix}. \qquad (33)
$$

It is easy to find the corresponding projection operators, and the Feynman-Stueckelberg propagator.

[5] Of course, the signs at the mass terms depend on, how do we associate the positive- or negative- frequency solutions with λ and ρ.

This is just related to the spin-parity basis rotation (unitary transformations). In the previous papers it was explained that the connection with the Dirac spinors is [17, 19].[6] For instance,

$$\begin{pmatrix}\lambda^S_\uparrow(\mathbf{p})\\ \lambda^S_\downarrow(\mathbf{p})\\ \lambda^A_\uparrow(\mathbf{p})\\ \lambda^A_\downarrow(\mathbf{p})\end{pmatrix} = \frac{1}{2}\begin{pmatrix}1 & i & -1 & i\\ -i & 1 & -i & -1\\ 1 & -i & -1 & -i\\ i & 1 & i & -1\end{pmatrix}\begin{pmatrix}u_{+1/2}(\mathbf{p})\\ u_{-1/2}(\mathbf{p})\\ v_{+1/2}(\mathbf{p})\\ v_{-1/2}(\mathbf{p})\end{pmatrix}, \tag{34}$$

provided that the 4-spinors have the same physical dimension. This represents itself the rotation of the spin-parity basis. However, it is usually assumed that the $\lambda-$ and $\rho-$ spinors describe the neutral particles, meanwhile $u-$ and $v-$ spinors describe the charged particles. Kirchbach [19] found the amplitudes for neutrinoless double beta decay ($00\nu\beta$) in this scheme. It is obvious from (34) that there are some additional terms comparing with the standard formulation.

One can also re-write the above equations into the two-component forms. Thus, one obtains the Feynman-Gell-Mann [18] equations.

Barut and Ziino [15] proposed yet another model. They considered γ^5 operator as the operator of the charge conjugation. Thus, the charge-conjugated Dirac equation has the different sign comparing with the standard formulation:

$$[i\gamma^\mu\partial_\mu + m]\Psi^c_{BZ} = 0\,, \tag{35}$$

and the so-defined charge conjugation applies to the whole system, fermion+electromagnetic field, $e \to -e$ in the covariant derivative. The superpositions of the Ψ_{BZ} and Ψ^c_{BZ} also give us the "doubled Dirac equation", similar to the equations for $\lambda-$ and $\rho-$ spinors. The concept of the doubling of the Fock space has been developed in the Ziino works (cf. [16, 20]) in the framework of the quantum field theory. In their case the charge conjugate states are the eigenstates of the chirality at the same time. Next, it is interesting to note that for the Majorana-like field operators we have

$$\left[\nu^{^{ML}}(x^\mu) + \mathcal{C}\nu^{^{ML\,\dagger}}(x^\mu)\right]/2 = \int \frac{d^3\mathbf{p}}{(2\pi)^3}\frac{1}{2E_p} \tag{36}$$

$$\sum_\eta\left[\begin{pmatrix} i\Theta\phi^{*\,\eta}_{_L}(p^\mu)\\ 0\end{pmatrix} a_\eta(p^\mu)e^{-ip\cdot x} + \begin{pmatrix}0\\ \phi^\eta_{_L}(p^\mu)\end{pmatrix} a^\dagger_\eta(p^\mu)e^{ip\cdot x}\right],$$

$$\left[\nu^{^{ML}}(x^\mu) - \mathcal{C}\nu^{^{ML\,\dagger}}(x^\mu)\right]/2 = \int \frac{d^3\mathbf{p}}{(2\pi)^3}\frac{1}{2E_p} \tag{37}$$

$$\sum_\eta\left[\begin{pmatrix}0\\ \phi^\eta_{_L}(p^\mu)\end{pmatrix} a_\eta(p^\mu)e^{-ip\cdot x} + \begin{pmatrix} -i\Theta\phi^{*\,\eta}_{_L}(p^\mu)\\ 0\end{pmatrix} a^\dagger_\eta(p^\mu)e^{ip\cdot x}\right],$$

which, thus, naturally lead to the Ziino-Barut scheme of massive chiral fields, Ref. [15].

[6] I also acknowledge personal communications from D. V. Ahluwalia on these matters.

Finally, I would like to mention that, in general, in the Weyl basis the $\gamma-$ matrices are *not* Hermitian, $\gamma^{\mu^\dagger} = \gamma^0\gamma^\mu\gamma^0$. So, $\gamma^{i^\dagger} = -\gamma^i$, $i = 1, 2, 3$, the pseudo-Hermitian matrix. The energy-momentum operator $i\partial_\mu$ is obviously Hermitian. So, the question, if the eigenvalues of the Dirac operator (the mass, in fact) would be always real? The question of the complete system of the eigenvectors of the *non*-Hermitian operator deserve careful consideration [21]. As mentioned before, Bogoliubov and Shirkov [11, p.55-56] used the scheme to construct the complete set of solutions of the relativistic equations, fixing the sign of $p_0 = +E_p$.

3. 4-Vector Field

The quantum field theory and the gauge theories had to give answers on many questins, including the unification. This did not happen. They have been successful in the description of scattering processes. Since the 60s I do not see any progress, except for the models which cannot be checked experimentally.

Let us repeat the textbook procedure how to construct field operators. During the calculations below we again have to present $1 = \theta(k_0) + \theta(-k_0)$ in order to get positive- and negative-frequency parts. However, one should be warned that in the point $k_0 = 0$ this presentation is ill-defined.

$$
\begin{aligned}
A_\mu(x) &= \frac{1}{(2\pi)^3} \int d^4k\, \delta(k^2 - m^2) e^{-ik\cdot x} A_\mu(k) = \qquad (38)\\
&= \frac{1}{(2\pi)^3} \sum_\lambda \int d^4k \delta(k_0^2 - E_k^2) e^{-k\cdot x} \epsilon_\mu(k,\lambda) a_\lambda(k) = \\
&= \frac{1}{(2\pi)^3} \int \frac{d^4k}{2E_k} [\delta(k_0 - E_k) + \delta(k_0 + E_k)][\theta(k_0) + \theta(-k_0)] e^{-ik\cdot x} A_\mu(k) = \\
&= \frac{1}{(2\pi)^3} \int \frac{d^4k}{2E_k} [\delta(k_0 - E_k) + \delta(k_0 + E_k)] \left[\theta(k_0) A_\mu(k) e^{-ik\cdot x} + \right.\\
&\left. + \theta(k_0) A_\mu(-k) e^{+ik\cdot x}\right] = \frac{1}{(2\pi)^3} \int \frac{d^3\mathbf{k}}{2E_k} \theta(k_0) [A_\mu(k) e^{+ik\cdot x} + A_\mu(-k) e^{+ik\cdot x}] = \\
&= \frac{1}{(2\pi)^3} \sum_\lambda \int \frac{d^3\mathbf{k}}{2E_k} [\epsilon_\mu(k,\lambda) a_\lambda(k) e^{-ik\cdot x} + \epsilon_\mu(-k,\lambda) a_\lambda(-k) e^{+ik\cdot x}]\,.
\end{aligned}
$$

Moreover, we should transform the second part to $\epsilon^*_\mu(k,\lambda) b^\dagger_\lambda(k)$ as usual. In such a way we obtain the charge-conjugate states. Of course, one can try to get P-conjugates or CP-conjugate states too. We set

$$
\sum_\lambda \epsilon_\mu(-k,\lambda) a_\lambda(-k) = \sum_\lambda \epsilon^*_\mu(k,\lambda) b^\dagger_\lambda(k)\,, \qquad (39)
$$

multiply both parts by $\epsilon_\nu[\gamma_{00}]_{\nu\mu}$, and use the normalization conditions for polarization vectors. γ_{00} is the metric tensor, in fact. It is the member of the covariant set $\gamma_{\mu\nu}$ The normalization condition is: $\epsilon^*_\mu(\mathbf{p},\sigma)\epsilon^\mu(\mathbf{p},\sigma) = -1$.

In the $(\frac{1}{2},\frac{1}{2})$ representation we can also expand (apart the equation (39)) in the different way:

$$\sum_\lambda \epsilon_\mu(-k,\lambda)a_\lambda(-k) = \sum_\lambda \epsilon_\mu(k,\lambda)a_\lambda(k)\,. \qquad (40)$$

From the first definition we obtain (the signs $\mp$ depends on the value of σ):

$$b_\sigma^\dagger(k) = \mp\sum_{\mu\nu\lambda}\epsilon_\nu(k,\sigma)[\gamma_{00}]_{\nu\mu}\epsilon_\mu(-k,\lambda)a_\lambda(-k)\,, \qquad (41)$$

or

$$b_\sigma^\dagger(k) =$$
$$= \frac{E_k^2}{m^2}\begin{pmatrix} 1+\frac{\mathbf{k}^2}{E_k^2} & \sqrt{2}\frac{k_r}{E_k} & -\sqrt{2}\frac{k_l}{E_k} & -\frac{2k_3}{E_k}\\ -\sqrt{2}\frac{k_r}{E_k} & -\frac{k_r^2}{\mathbf{k}^2} & -\frac{m^2k_3^2}{E_k^2\mathbf{k}^2}+\frac{k_rk_l}{E_k^2} & \frac{\sqrt{2}k_3k_r}{\mathbf{k}^2}\\ \sqrt{2}\frac{k_l}{E_k} & -\frac{m^2k_3^2}{E_k^2\mathbf{k}^2}+\frac{k_rk_l}{E_k^2} & -\frac{k_l^2}{\mathbf{k}^2} & -\frac{\sqrt{2}k_3k_l}{\mathbf{k}^2}\\ \frac{2k_3}{E_k} & \frac{\sqrt{2}k_3k_r}{\mathbf{k}^2} & -\frac{\sqrt{2}k_3k_l}{\mathbf{k}^2} & \frac{m^2}{E_k^2}-\frac{2k_3}{\mathbf{k}^2}\end{pmatrix}\begin{pmatrix} a_{00}(-k)\\ a_{11}(-k)\\ a_{1-1}(-k)\\ a_{10}(-k)\end{pmatrix}.$$

From the second definition $\Lambda^2_{\sigma\lambda} = \mp\sum_{\nu\mu}\epsilon^*_\nu(k,\sigma)[\gamma_{00}]_{\nu\mu}\epsilon_\mu(-k,\lambda)$ we have

$$a_\sigma(k) = \begin{pmatrix} -1 & 0 & 0 & 0\\ 0 & \frac{k_3^2}{\mathbf{k}^2} & \frac{k_l^2}{\mathbf{k}^2} & \frac{\sqrt{2}k_3k_l}{\mathbf{k}^2}\\ 0 & \frac{k_r^2}{\mathbf{k}^2} & \frac{k_3^2}{\mathbf{k}^2} & -\frac{\sqrt{2}k_3k_r}{\mathbf{k}^2}\\ 0 & \frac{\sqrt{2}k_3k_r}{\mathbf{k}^2} & -\frac{\sqrt{2}k_3k_l}{\mathbf{k}^2} & 1-\frac{2k_3^2}{\mathbf{k}^2}\end{pmatrix}\begin{pmatrix} a_{00}(-k)\\ a_{11}(-k)\\ a_{1-1}(-k)\\ a_{10}(-k)\end{pmatrix}. \qquad (42)$$

It is the strange case: the field operator will only destroy particles. Possibly, we should think about modifications of the Fock space in this case, or introduce several field operators for the $(\frac{1}{2},\frac{1}{2})$ representation as in the $(1,0)\oplus(0,1)$ rep.

4. Conclusions.

The main points of my paper are: there are "negative-energy solutionsïn that is previously considered as "positive-energy solutions.ºf relativistic wave equations, and vice versa. Their explicit forms have been presented in the case of spin-1/2. Next, the relations to the previous works have been found. For instance, the doubling of the Fock space and the corresponding solutions of the Dirac equation have additional mathematical bases in this paper. Similar conclusion can be deduced for higher-spin equations.

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Referencias

[1] Guang-jiong Ni, In *Relativity, Gravitation, Cosmology.* (Nova Science Pub., Huntington, NY, USA, 2004), hep-ph/0306028.

[2] N. Simicevic, *Implications of Weak-Interaction Space Deformation for Neutrino Mass Measurements*, nucl-th/9909034.

[3] S. M. Bilenky and C. Giunti, *Nucl. Phys. Proc. Suppl.* **48**, 381 (1996).

[4] A. O. Barut, *Phys. Lett.* B**73**, 310 (1978); *Phys. Rev. Lett.* **42**, 1251 (1979).

[5] D. V. Ahluwalia and A. Ch. Nayak, *Int. J. Mod. Phys* D**23**, 1430026 (2014).

[6] V. V. Dvoeglazov, *Adv. Appl. Clifford Algebras* **26**, 1173 (2016).

[7] P. A. M. Dirac, *Proc. Roy. Soc. Lond.* A**117**, 610 (1928).

[8] J. J. Sakurai, *Advanced Quantum Mechanics.* (Addison-Wesley, 1967).

[9] L. H. Ryder, *Quantum Field Theory.* (Cambridge University Press, Cambridge, 1985).

[10] C. Itzykson and J.-B. Zuber, *Quantum Field Theory.* (McGraw-Hill Book Co., 1980), p. 156.

[11] N. N. Bogoliubov and D. V. Shirkov, *Introduction to the Theory of Quantized Fields.* 2nd Edition. (Nauka, Moscow, 1973).

[12] V. V. Dvoeglazov, *J. Phys. Conf. Ser.* **284**, 012024 (2011), arXiv:1008.2242.

[13] V. V. Dvoeglazov, *Hadronic J. Suppl.* **18**, 239 (2003), physics/0402094; *Int. J. Mod. Phys.* B**20**, 1317 (2006).

[14] M. Markov, *ZhETF* **7**, 579 (1937); ibid. 603; *Nucl. Phys.* **55**, 130 (1964).

[15] A. Barut and G. Ziino, *Mod. Phys. Lett.* A**8**, 1099 (1993); G. Ziino, *Int. J. Mod. Phys.* A **11**, 2081 (1996).

[16] I. M. Gelfand and M. L. Tsetlin, *ZhETF* **31** 1107 (1956); G. A. Sokolik, *ZhETF* **33** , 1515 (1957).

[17] V. V. Dvoeglazov, *Int. J. Theor. Phys.* **34**, 2467 (1995); *Nuovo Cim.* **108**A, 1467 (1995); *Hadronic J.* **20**, 435 (1997); *Acta Phys. Polon.* B**29**, 619 (1998).

[18] R. P. Feynman and M. Gell-Mann, *Phys. Rev.* **109**, 193 (1958).

[19] M. Kirchbach, C. Compean and L. Noriega, *Eur. Phys. J.* A**22**, 149 (2004).

[20] V. V. Dvoeglazov, *Int. J. Theor. Phys.* **37**, 1015 (1998).

[21] V. A. Ilyin, *Spektralnaya Teoriya Differencialnyh Operatorov.* (Nauka, Moscow, 1991); V. D. Budaev, *Osnovy Teorii Nesamosopryazhennyh Differencialnyh Operatorov.* (SGMA, Smolensk, 1997).

[22] S. S. Schweber, *An Introduction to Relativistic Quantum Field Theory.* (Row, Peterson and Co, Elmsford, 1961).

[23] Kerson Huang, *Quantum Field Theory: From Operators to Path Integrals.* (Wiley, NY, 1998).

[24] S. S. Schweber and G. Sudarshan, *Ann. Phys.* **19**, 351 (1962).

[25] T. De Graaf and H.A. Tolhoek, *Phase factors in discrete symmetry operators in quantum field theory. Physica* **43**, 142 (1969).

[26] V. V. Dvoeglazov, *Int. J. Mod. Phys.* B**20**, 1317 (2006); *Hadronic J. Suppl.* **18**, 239 (2003).

[27] M. N. Mnatsakanova and Yu. S. Vernov, arXiv:0901.1454v1.

[28] V. V. Dvoeglazov, *Hadronic J.* **20**, 435 (1997).