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A Theory of Mass and Energy with Reference to Surfing Momentum[1]

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Abstract

Based on a relativistic model for a quantum particle, the paper proposes a new definition of energy which takes into account the additional localised surfing motion of the particle on its phase surface which is perpendicular to its translational motion. The conventional relativistic formula for energy is found to be a limiting case for this new definition of energy when the characteristic parameter in the new energy formula is set to zero. For a particle at rest translationally, the two formulae yield the same energy, but in general for the same non-zero translational velocity they yield different energies, the difference being dependent on the value of the characteristic parameter. A certain kinetic energy range has been theoretically identified where the performances of the conventional formula and the new formula of energy can be best tested against experimental data. The new definition of energy prompts a definition of rest mass, m, which is the surfing momentum of the particle on its phase surface divided by c. The surfing energy of the particle is its surfing momentum multiplied by c. The surfing energy is therefore given by mc^2 which has been conventionally understood as the rest energy. The theoretical analysis therefore shows from the new definition of energy how the link between surfing momentum and mass, and the link between surfing momentum and surfing energy (rest energy), lead to the well known relationship between rest mass and rest energy, without invoking electromagnetism. Surfing energy, though identical to rest energy, affords a more physically intuitive understanding than rest energy because it can be visualised. Surfing energy, validly understood as an internal kinetic energy of the particle, can be converted into other forms of energy including photon energy, which is accompanied by a reduction of the particle's rest mass due to a simultaneous reduction in its surfing momentum. The notion of conversion of matter or mass into energy is critically interrogated because of their fundamental difference in dimension. It may be better to speak of the conversion of internal surfing energy into other forms of energy. This has implications for our understanding of nuclear reaction, the processes known as annihilation and creation of particle pairs, and dark matter.

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[1] This paper is dedicated to my late parents, Mr. Chun Loy So and Mrs. Sui Tai So, who worked hard in difficult circumstances to bring up my brothers and myself. An earlier version can be found at https://web.ma.utexas.edu/mp_arc/c/21/21-39.pdf. This revised version identifies the appropriate kinetic energy range for experimental verification of the proposed theory. It also probes further the relationship between mass and energy.

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1.0 Introduction

Physicists have been looking for the dark matter necessary to hold the galaxies together gravitationally. Also, dark energy is necessary to account for the increasing inflation rate of the universe. As dark matter can also be considered in terms of energy, this energy and the dark energy are the two kinds of energy which have been elusive to physicists. In view of this, any useful additional insight into the nature of mass and the nature of energy should be welcome.

Hestenes wrote a paper on 'The *Zitterbewegung* Interpretation of Quantum Mechanics' [1] which encouraged other physicists such as Salesi, Recami and Espositio to write papers [2,3,4] along that line. These papers give some indication of what particle spin could be. Hestenes hypothesised in Section 4 of his paper, 'The so-called "rest mass" of the electron is therefore a kinetic energy of the magnetic self-interaction. It is this that gives the electron its inertial properties.' He also referenced the 'flywheel-like nature of this inertia' but he did not investigate the detailed dynamics of such inertia in that paper. More recently, he again suggested in his essay on 'Electron time, mass and zitter' [5], 'The spin-zitter hypothesis has implications for gravitational fields as well as sources. It tells us that there is no mass without spin.' In a paper on 'Spin and Relativity', Lepadatu [6] also hypothesised, 'The inertia is an intrinsic property due to the spin motion of the particles, ...' Rockenbauer [7] referenced Hestenes and Lepadatu in his paper which is entitled 'Can the spinning of elementary particles produce the rest energy mc^2? The vortex model of elementary particles'. He also hypothesised, '[T]he rest energy can be produced in full by the spinning motion of elementary particles if the peripheral speed is equal to the velocity of light.' The challenge with these hypotheses is that they rely on the particle moving at the speed of light. This paper will take a significantly different route to the routes indicated by these authors and will not require a particle to move at the speed of light. It will make use of the idea that the rest mass is generated from the inherent mass of a particle by some suitable surfing motion on the phase surface – this surfing motion incorporates the spinning motion of the particle but has an additional velocity component. Hence, contrary to Rockenbauer, the paper will suggest that the surfing energy of the particle on the phase surface, which includes but is greater than its spin energy, is responsible for the rest energy of the particle.

The paper will briefly introduce the relativistic model for a particle and its spin which has been given extensive treatment by the author in a previous paper [8]. The focus of this paper is on the implications of the relativistic model for our understanding of the mass and energy of a particle. Briefly, in the model, because of the additional surfing motion of the particle on its phase surface which is perpendicular to its translational motion, the total speed of a particle is always greater than its translational speed. This implies that the Lorentz factor in the model is always greater than the one conventionally given merely by the translational speed. Hence, the conventional expression for the energy of a particle, given by the smaller conventional Lorentz factor, invariably yields an energy below that given by the larger Lorentz factor in the model adopted here (see later). A new definition of energy will be presented incorporating the larger Lorentz factor. It will be shown that the conventional relativistic formula for energy is a limiting case for this new definition of energy. The formula corresponding to the new definition of energy can be verified or falsified by experimental data. A certain kinetic energy range will be identified where the new formula of energy can be best tested against experimental data. If the new definition is verified, this will have implications for our understanding of mass, energy, dark matter and other matters in physics.

A puzzling question concerning the so-called equivalence of mass and energy is this: what is the mechanism for converting mass into energy if indeed such conversion explicitly happens? Or are there other ways for explaining such ‘equivalence’? We know from nuclear fission and nuclear fusion that energy is related to mass. However, it is not so clear exactly how the conversion of mass into energy, and vice versa, take place, given that they have different physical dimensions. The new definition of energy, which involves the notions of surfing momentum and surfing energy, could shed light on this puzzling question.

2.0 Surfing Motion of a Particle on Its Phase Surface

The relativistic model for a quantum particle is given in Section 8 of [8]. Here it is presented in a highly summarised form. A particle has three momentum components:

$$\underset{\sim}{p}_1 \equiv \hbar \nabla S \ , \ \underset{\sim}{p}_2 \equiv \lambda_2 \frac{\hbar}{2} \frac{\nabla \rho}{\rho} \wedge \vec{s} \ , \ \underset{\sim}{p}_3 \equiv \lambda_3 \hbar \frac{\nabla S}{\rho} \wedge \vec{a}$$

where the wave function is written as $\psi = Re^{iS}$, $\rho \equiv R^2$ is the probability density, $\vec{s}$ at the particle's position is a unit vector perpendicular to $\nabla\rho$ and lies on the plane formed by ∇S and $\nabla\rho$, $\vec{a}$ is the unit vector in the direction of $\underset{\sim}{p}_2$ at the point where the particle is, λ_2 is a constant corresponding to the spin number of the particle in question, λ_3 is constant over space but varies in time non-deterministically. The three momentum components form an orthogonal set of vectors and they correspond to the three velocity components, $\underset{\sim}{v}_1, \underset{\sim}{v}_2, \underset{\sim}{v}_3$, which also form a set of orthogonal vectors. $\underset{\sim}{v}_1$ is called the translational velocity. $\underset{\sim}{v}_2$ and $\underset{\sim}{v}_3$ lie on a plane tangential to the *S* (phase) surface at the position where the particle is. This means that $\underset{\sim}{v}_s \equiv \underset{\sim}{v}_2 + \underset{\sim}{v}_3$ is a velocity on that tangential plane so that the particle can be said to be surfing on the *S* surface while moving forward with translational velocity $\underset{\sim}{v}_1$. Hence, $\underset{\sim}{v}_s$ is called the surfing velocity.

It will be instructive to illustrate these three orthogonal velocity components with the case of a free particle with no slit in its path to diffract it. The following Helmholtz equation in *R* applies both in the relativistic framework and the non-relativistic framework:

$$\nabla^2 R + a^2 R = 0$$

where *a* is a constant.[3] (This implies that in free space with no slit, the quantum potential, $-\frac{\hbar^2}{2m}\frac{\nabla^2 R}{R}$ is constant.) In a system of cylindrical co-ordinates, (*r, θ, z*), we adopt the convention that $\underset{\sim}{v}_1$ is in the *z* direction. It can be easily shown that a *S* surface is then identical to the (*r, θ*) plane (or the *z* plane) on which the particle surfs non-deterministically. It can also be shown that the *R* surfaces, and therefore the *ρ* surfaces, are circular tubes extending along the direction of *z*; and *R*=0 at a certain distance, *L*, from the centre.[4] Some sample circular *R* contours and the surfing velocity components, $\underset{\sim}{v}_2$ and $\underset{\sim}{v}_3$, are illustrated in Figure 1. Again, $\underset{\sim}{v}_3$ varies non-deterministically but will be subjected to the bulk statistical constraint of Born's rule (see below).

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[3] See (§7) in [8] for the non-relativistic case and (§22) for the relativistic case.

[4] See section 4.0 of [8] for the mathematical details which also apply to the Helmholtz equation of the same form for the relativistic case, i.e., (§22) of [8].

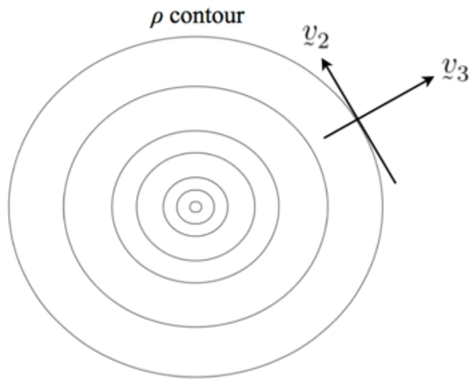

Figure 1: $\underset{\sim}{v}_2$ and $\underset{\sim}{v}_3$ on the (r, θ) plane (or S surface); the z direction and hence the direction of $\underset{\sim}{v}_1$ is perpendicular to the page

The choices of the forms of the three momentum components are justified by correspondence to observational data as follows. Firstly,

$$\left|\underset{\sim}{p}_1\right| \equiv \hbar|\nabla S| = \hbar\frac{2\pi}{\lambda} = \frac{h}{\lambda}$$

reproduces de Broglie's formula relating the momentum of a particle to the 'wavelength' of a particle. Hence, the choice of $\underset{\sim}{p}_1$ is consistent with (or vindicated by) the *observed* de Broglie's relation. Secondly, in the free particle case, integrating the angular momentum, given by the product of $\underset{\sim}{p}_2$ (the spinning momentum) and the radial distance, over the domain where the particle can be found yields $\lambda_2\hbar$, which is the angular momentum of the particle and is constant. Hence, the choice of the expression for $\underset{\sim}{p}_2$ is consistent with (or vindicated by) the *observed* constant angular momentum of a particle (see section 8.3.1 of [8] for details). Thirdly, $\underset{\sim}{p}_3$ is prescribed as the non-deterministic momentum through the non-deterministic λ_3 in such a way as to satisfy (i) the *observed* Born's rule (thus limiting the particle within the relevant finite localised domain, $r < L$, with $\rho(L) = 0$) and (ii) the *observed* non-determinacy of a particle. Hence, all three prescribed momenta match the observations in our universe and in that sense they are credible. In the philosophy of science, a theory's credibility is best assessed by its correspondence to observation, even though the criterion of elegance can be a supplementary criterion (to some degree the latter is true for the set of three orthogonal momenta). In the sense of correspondence to observation and in the sense of elegance, the three prescribed momenta are credible and will be used in this paper.[5]

Corresponding to the surfing velocity on the *S* surface, $\underset{\sim}{v}_s$, the surfing momentum is defined as

$$\underset{\sim}{p}_s \equiv \underset{\sim}{p}_2 + \underset{\sim}{p}_3$$

which will be a significant entity in our consideration of rest mass and rest energy.

3.0 'Rest Mass', 'Rest Energy', Einstein's Energy Formula and a New Energy Formula

Einstein's well known energy formula is

$$E = mc^2$$

which can be applied to a particle with zero translational velocity. Note that, this formula does not involve the notion of a surfing momentum as this paper suggests. Hence, this formula assumes that the particle is completely at rest, *m* is the rest mass and the energy, *E*, is the rest energy.

For non-zero translational velocity with speed v_1, *E* can be written as

$$E = m_r c^2$$

where m_r is the relativistic mass, $m_r = m/\sqrt{1 - v_1^2/c^2}$. In this paper, we call this energy formula as Einstein's generalised energy formula, or simply as Einstein's formula. This energy formula can be re-written as

[5] See section 10.1 of [8] for a summary of how these three velocity components are consistent with a significant number of other observed phenomena which thus add credibility to them.

$$E^2 = p_{1E}^2 c^2 + m^2 c^4 \tag{§1}$$

where p_{1E} is the magnitude of the momentum as Einstein envisaged it:

$$p_{1E} \equiv \frac{m v_1}{\sqrt{1 - \frac{v_1^2}{c^2}}} \tag{§2}$$

where again v_1 is the translational speed and the surfing motion perpendicular to $\underset{\sim}{v}_1$ is not involved in the expression for the magnitude of this momentum which is thus called the translational momentum. These expressions for the magnitude of the momentum and the energy are the conventional expressions. The subscript 'E' in p_{1E} signifies its expression according to Einstein and the subscript '1' signifies that it is the translational momentum.

If the surfing motion and the associated surfing momentum of the particle in the model introduced in Section 2 are to be represented and included in the expression for energy in (§1), how is this inclusion possible? Since the first term on the r.h.s. of (§1) comes from the translational momentum, it is logical that the surfing momentum could be related to the second term in (§1), m^2c^4 . And if we set the magnitude of the surfing momentum as $p_s = mc$, then (§1) becomes $E^2 = p_{1E}^2 c^2 + p_s^2 c^2$ .This looks like a balanced expression with the translational momentum and the surfing momentum both contributing to the energy. Furthermore, if we denote the magnitude of the total momentum by p, then $p^2 = p_{1E}^2 + p_s^2$ and $E = pc$. But now since the total speed, which includes the surfing speed, is no longer merely v_1, the Lorentz factor for the translational momentum should no longer be that in (§2). Furthermore, the definition of the magnitude of the surfing momentum, p_s(see §6), can involve a mass parameter which is not necessarily the same as the rest mass m, that is, it is possible that the rest mass, $m = p_s/c$, is generated by p_s which incorporates a different and a more fundamental mass in its expression. The larger Lorentz factor and the possibility of a more fundamental mass suggest that we should begin with a fresh and a more radical basis which will give consistency and elegance to the forms of translational momentum, surfing momentum and energy. Nevertheless, the above intuitive exercise has led us to see the possible relationship between the surfing momentum and rest mass. (For the sake of brevity, from now on whenever the term ‘momentum’ is used, unless otherwise stated, it means the magnitude of the corresponding momentum.) Now, we define a new energy with subscript *N* (for new) to distinguish it from *E* given by (§1), a new total momentum, p, and a new translational momentum, p_{1N}:

$$E_N \equiv pc \tag{§3}$$

$$p \equiv \frac{m_i v}{\sqrt{1-\frac{v^2}{c^2}}} \tag{§4}$$

$$p_{1N} \equiv \frac{m_i v_1}{\sqrt{1-\frac{v_1^2+v_s^2}{c^2}}} \tag{§5}$$

where v is the total speed, $p = \left|\underset{\sim}{p}\right|$, $\underset{\sim}{p} = \underset{\sim}{p}_{1N} + \underset{\sim}{p}_s$, $\underset{\sim}{p}_s = \underset{\sim}{p}_2 + \underset{\sim}{p}_3$, m_i is the ‘inherent mass’ of the particle. The term ‘inherent mass’ means the mass inherent to the particle which is not generated from any motion of the particle, not even its surfing motion on the *S* surface. $\underset{\sim}{p}$ is the total momentum (vector) of the particle taking into account both the translational momentum *and* the surfing momentum on the *S* surface. Similar to the total momentum in (§4) and the translational momentum in (§5), the surfing momentum is consistently defined as

$$p_s \equiv \frac{m_i v_s}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{m_i v_s}{\sqrt{1-\frac{v_1^2+v_s^2}{c^2}}} \tag{§6}$$

where v_s is the surfing speed in general. These definitions of energy and momenta consistently use the same Lorentz factor which include the surfing speed, v_s, and the same inherent mass, m_i. Taking the square of (§3), we have

$$E_N^2 = p^2c^2 = p_{1N}^2c^2 + p_s^2c^2 \ . \qquad (§7)$$

If we compare (§7) with (§1), we see that m^2c^4 could be equivalent to and therefore accounted for by $p_s^2c^2$ such that

$$p_s = mc \qquad (§8)$$

The 'rest mass', $m = p_s/c$, is therefore generated by the surfing momentum. In normal circumstances, a constant *m* requires a constant p_s but *m* could vary with a varying p_s in exceptional circumstances (see later). Since the surfing momentum on the *S* surface is perpendicular to the translational momentum, the variation in the translational motion of a particle should not affect the magnitude of the surfing momentum and thus the rest mass. This is reasonable. As the translational speed v_1 varies, the total speed v varies, v_s will need to adjust to maintain a constant p_s according to (§6); see §13 below.

In the particular case when the translational speed is zero, v_{so} is the surfing speed and

$$p_s = \frac{m_i v_{so}}{\sqrt{1 - \frac{v_{so}^2}{c^2}}} \qquad (§9)$$

The generated 'rest mass' , which is p_s / *c,* can be set by using this expression for zero translational speed. At this point it may be good to introduce the term, 'effective mass', in addition to 'rest mass' which can be somewhat misleading since the particle is not at rest due to the surfing motion on the *S* surface even when its translational speed is zero. From now on the term 'effective mass' and the term 'rest mass', denoted by *m*, will have the same meaning which is nevertheless different from the meaning of 'inherent mass', m_i. Again for zero v_1, both p_{1E} and p_{1N} are zero; using (§7) and (§8),

$$E_N = p_s c = E = mc^2$$

which is normally called the 'rest energy'. But since the particle is not genuinely at rest with its surfing motion, in this paper this is also called the 'surfing energy' (because the energy is in the surfing motion) or 'base energy' (because it is the basic energy for zero translational motion).

Note that, for zero $\underset{\sim}{v}_s$ and therefore zero p_s, the effective mass, *m*, will be zero even though its inherent mass, m_i , is not zero. This brings to mind 'massless' particles whose effective mass is considered to be zero. In the model adopted *in this paper, these particles are 'massless' in the sense that their effective mass is zero, but zero effective mass does not exclude the possibility of these 'massless' particles having non-zero inherent mass*. The notion of 'massless' particle could be useful in understanding particle and anti-particle annihilation where the energy released in the annihilation is twice the surfing energy (rest energy) of the two particles. This could be understood in terms of the particle and anti-particle giving up their surfing momentum and their associated surfing energy – which is twice the surfing energy (rest energy, $p_s c = mc^2$) – and becoming a joint 'massless' entity (with zero effective mass) which nevertheless has inherent mass. This will be considered further in the Section 5 on Discussion. Also, according to (§5), such a 'massless' entity can have non-zero translational momentum if its inherent mass and its translational speed are non-zero; and it can have non-zero energy which is given by

$$E_N = p_{1N} c \qquad (\S 10)$$

according to (§7). The notion of non-zero energy for a 'massless' entity formed from annihilation could, at least partially, account for the elusive dark matter and will be discussed further also in the section on Discussion.

4.0 Comparison Between Einstein's Energy Formula, the New Energy Formula with Possible Experimental Verification

If v_1 is greater than zero,

$$\frac{p_{1E}}{p_{1N}} = \frac{m}{m_i}\frac{\sqrt{1-\frac{v_1^2+v_s^2}{c^2}}}{\sqrt{1-\frac{v_1^2}{c^2}}} = \frac{m}{m_i}\frac{\sqrt{c^2-(v_1^2+v_s^2)}}{\sqrt{c^2-v_1^2}}$$

where it can be seen that p_{1N} will tend to be larger than p_{1E} when v_s is greater than zero. This is due to the crucial fact that the Lorentz factor in p_{1N} takes into account the effect of the surfing velocity on the *S* surface while the Lorentz factor in p_{1E} does not. The Lorentz factor in p_{1N} is naturally greater than the Lorentz factor in p_{1E} which lacks the surfing speed in its definition. However, the ratio between the p_{1E} and p_{1N} also depends on the ratio between the effective mass and the inherent mass. Now,

$$\frac{m_i}{m} = \frac{m_i c}{p_s} = \frac{m_i\sqrt{c^2-v^2}}{m_i v_s} = \frac{\sqrt{c^2-v_{so}^2}}{v_{so}} . \qquad (\S 11)$$

For $v_{so} = \frac{c}{\sqrt{2}},\ m = m_i.$

For $v_{so} > \frac{c}{\sqrt{2}}\ ,\ m > m_i.$

For $v_{so} < \frac{c}{\sqrt{2}}\ ,\ m < m_i.$

The ratio between the p_{1E} and p_{1N} will affect the the ratio between E and E_N . Since m^2c^4 of (§1) is identical to $p_s^2c^2$ of (§7), the difference between E^2 and E_N^2 lies in the magnitudes of p_{1E} and p_{1N}. If p_{1N} is greater than p_{1E}, then E_N will be greater than *E*. In that case, since E_N and *E* have the same surfing energy or rest energy, the kinetic energy in E_N, *defined as the difference between the total energy* E_N *and the surfing energy*, will be greater than the kinetic energy in *E*. To investigate whether this is the case, we will plot the graphs relating the kinetic energy of a particle to its translational speed for various values of

$\frac{m_i}{m}$. Before we can do that, we need to work out the expressions for the kinetic energy in E_N and the kinetic energy in *E*.

For the Einstein case, the particle's kinetic energy is $E - mc^2 = m_r c^2 - mc^2$ where $m_r = m/\sqrt{1 - v_1^2/c^2}$ is the relativistic mass. It can be easily shown that, the kinetic energy (written as *K.E.*), when normalised with respect to its rest energy, can be written as

$$\frac{K.E.}{mc^2} = \left(1 - \frac{v_1^2}{c^2}\right)^{-\frac{1}{2}} - 1 \quad (\S 12a)$$

where it can be seen that the first term on the r.h.s. is the Lorentz factor.

For the case of the new definition of energy, the particle's kinetic energy (*K.E.*) due to its translational motion is defined as the total energy minus the surfing energy (rest energy),

$$K.E. \equiv E_N - mc^2 = E_N - p_s^2/m \quad (\S 12b)$$

which, when normalised with respect to its surfing energy, is

$$\frac{K.E.}{mc^2} = \frac{E_N}{mc^2} - 1 = \left(\frac{p_{1N}^2 c^2 + p_s^2 c^2}{m^2 c^4}\right)^{\frac{1}{2}} - 1 = \left(\frac{p_{1N}^2}{p_s^2} + 1\right)^{\frac{1}{2}} - 1$$

where $\frac{p_{1N}^2}{p_s^2} = \frac{(v_1/c)^2}{(v_s/c)^2}$ according to (§5) and (§6). Using (§6) and (§8),

$$\left(\frac{v_s}{c}\right)^2 = \left(1 - \frac{v_1^2}{c^2}\right) \Big/ \left(1 + \frac{m_i^2}{m^2}\right) \quad (\S 13)$$

which incidentally shows that $\left(\frac{v_s}{c}\right)^2$ decreases linearly with $\frac{v_1^2}{c^2}$ and approaches zero as $\frac{v_1^2}{c^2}$ approaches 1. Using the above relationships,

$$\frac{K.E.}{mc^2} = \left(1 + \frac{\frac{v_1^2}{c^2}\left(1 + \frac{m_i^2}{m^2}\right)}{1 - \frac{v_1^2}{c^2}}\right)^{\frac{1}{2}} - 1 \quad (\S 14)$$

where $\frac{m_i^2}{m^2}$ is an additional parameter that (§12a) does not have in Einstein's case. It can be seen that when $\frac{m_i^2}{m^2}$ is zero, (§14) reduces to (§12a), i.e., zero $\frac{m_i^2}{m^2}$ corresponds to Einstein's case. But other than this special case, $\frac{m_i^2}{m^2}$ is non-zero.

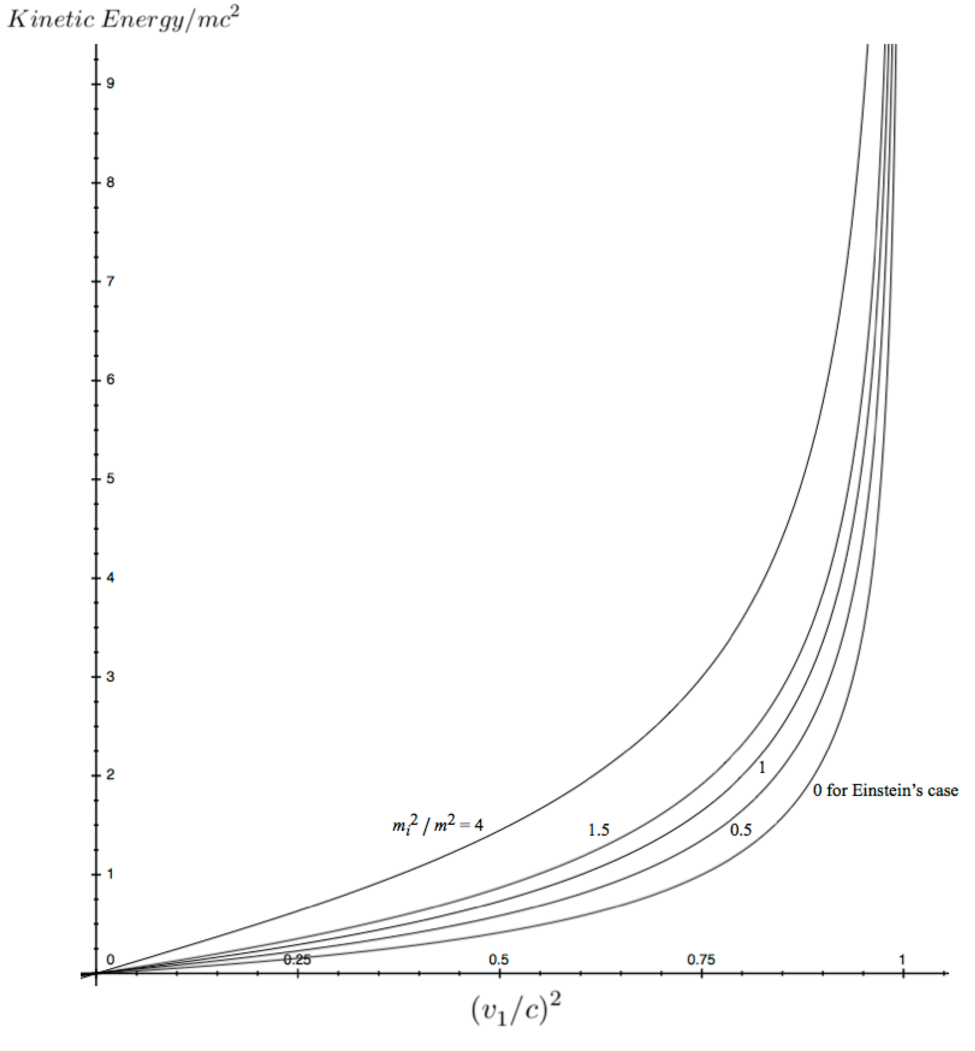

Figure 2: $Kinetic\ Energy/mc^2$ vs $(v_1/c)^2$ with parameter value for $m_i^2\ /\ m^2$ as 4, 1.5, 1, 0.5 and 0 (Einstein's formula)

It is readily seen that as $m_i^2\ /\ m^2$ increases from zero, the curve shifts to the left in Figure 2. This means that for a given kinetic energy of a particle, the translational velocity according to the new formula will be slower than that according to Einstein's formula. Equivalently, for any given translational velocity, the kinetic energy according to Einstein's formula is invariably less than the kinetic energy (due to the translational motion, not the surfing motion) according to the new formula; and the difference in kinetic energy between the two formulae for a given translational speed, $\triangle(Kinetic\, Energy/mc^2)$, increases with increasing value of $m_i^2\ /\ m^2$. Also, for a given parameter value of $m_i^2\ /\ m^2$, the difference in kinetic energy between the two formulae will tend to infinity as the translational speed, v_1, approaches c, as can be seen in the following graph. It is possible that different particles may have different values for $m_i^2\ /\ m^2$.

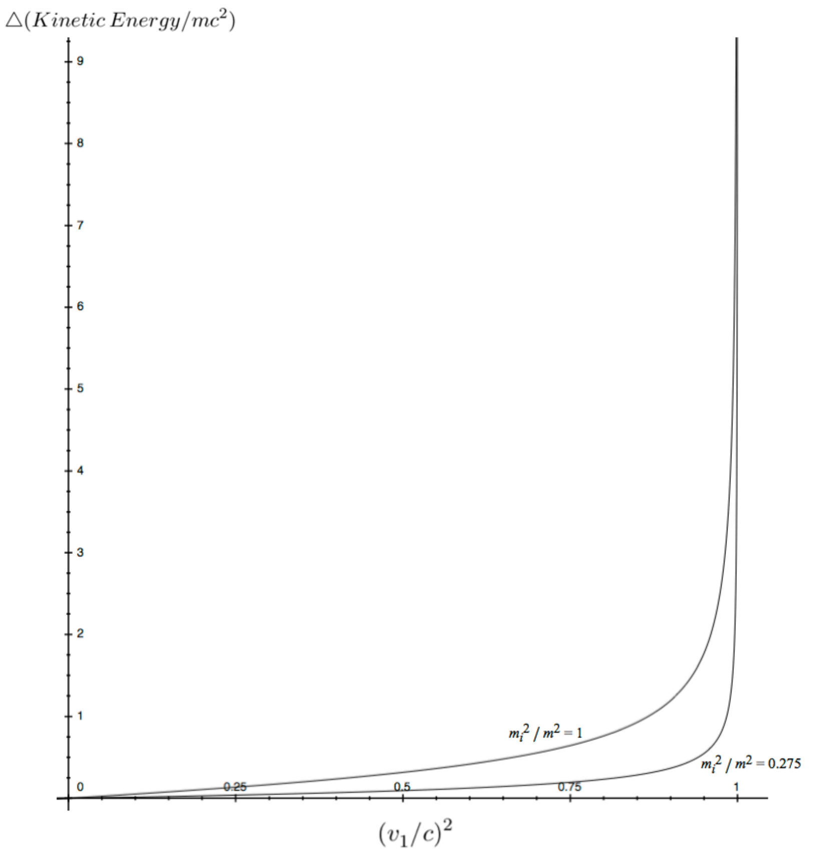

Figure 3: Difference in kinetic energy (normalised by rest mass) between Einstein's formula and the new formula , $\triangle(Kinetic\, Energy/mc^2)$, vs $(v_1/c)^2$, with parameter value for $m_i^2\ /\ m^2$ of the new formula as 1 and 0.275.

It will be interesting to see from experimental data which formula is closer to reality. For this purpose, it may appear that since the kinetic energy difference between the two formulae is greatest when the velocity is closest to *c*, one should look at the data with very high velocity and very high kinetic energy but this may not be the case. For a given very high kinetic energy, the velocities predicted by the two formulae are very close to one another (see Figure 2). Because of the possible error in measurement, a measured velocity closer to the velocity predicted by one formula than the velocity predicted by the other formula does not necessarily means that the former formula is closer to reality than the latter formula. For example, it is possible that the real velocity value associated with one formula (which happens to be the reality) may be distorted in the process of measurement so that its measured value is shifted from the curve corresponding to that formula (reality) and lands on the curve for the other formula (non-reality). Because the velocities predicted by the two formulae are very close to one another at very high kinetic energy, to ascertain which formula corresponds to reality (or is closer to reality) at very high kinetic energy will require very high measurement accuracy for the velocity which may be too difficult to achieve experimentally. An example can be seen in the data produced by Bertozzi [9] who carried out experiments at MIT with electrons in the 1960s to verify Einstein's formula. Figure 4 gives the plot of four data points (out of his five points) with reference to Einstein's formula (and Newton's formula).[6]

[6] The last data point from Bertozzi is not used in the plot. For that data point, the kinetic energy is so large that the translational speed is very close to *c* such that he set it to be equal to *c*. But that requires infinite kinetic energy, not a finite large kinetic energy. This means that accurately measuring the translational speed so close to *c* is beyond the capacity of his experiment. Hence, that data point is not used in the plot in Figure 4.

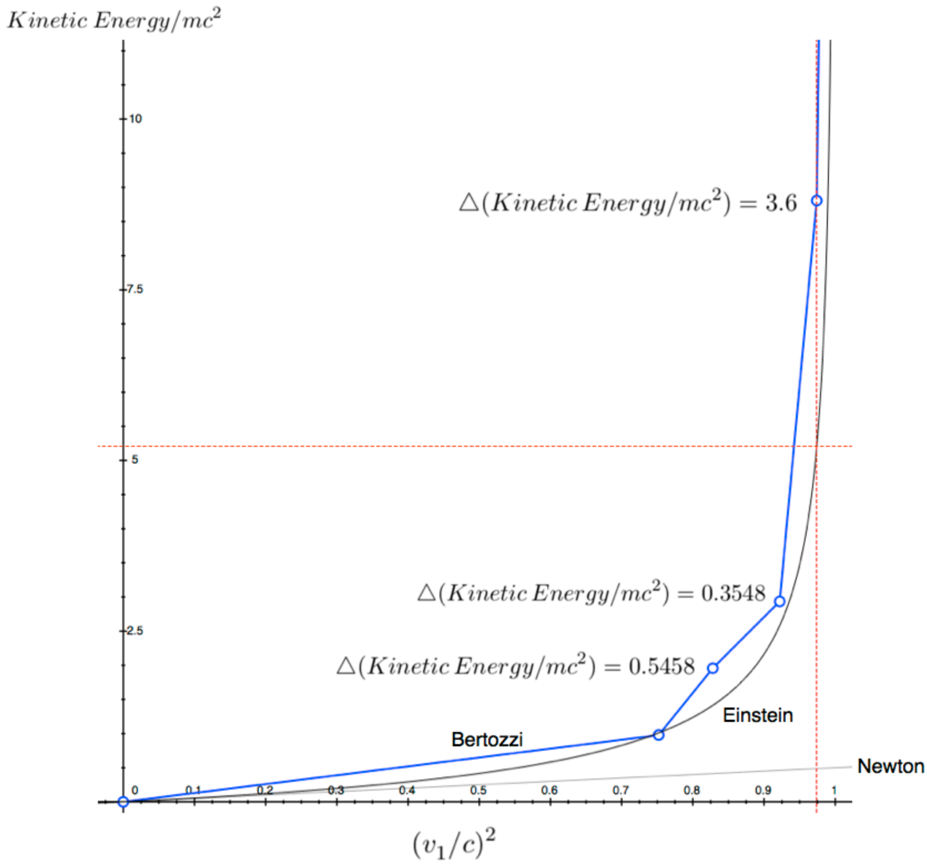

Figure 4: $Kinetic\ Energy/mc^2$ vs $(v_1/c)^2$ for Einstein, Bertozzi and Newton

Bertozzi's experimental data follow the general trend of the curve for Einstein's formula but apart from the data point with the lowest kinetic energy, the measured velocities for the higher kinetic energies are all slower than the values predicted by Einstein's formula. Furthermore, if by $\triangle(Kinetic\ Energy/mc^2)$ we now denote the difference between the kinetic energy in a data point from Bertozzi and the kinetic energy given by Einstein's formula, the greatest $\triangle(Kinetic\ Energy/mc^2)$ marked by the red dotted lines in Figure 4 is 3.6 times of the electron's rest energy (or surfing energy) which is a very large amount of energy for the electron. All these seems to be consistent with the prediction of the new formula. However, it is possible that Einstein's formula still corresponds to reality since a small error in measuring the velocity can account for the slight shift of the velocity from the value predicted by Einstein's formula to the measured value which thus accounts for the large $\triangle(Kinetic\ Energy/mc^2)$. On the other hand, it is possible that the new formula with a certain value for the parameter, $m_i^2 \,/\, m^2$, corresponds to reality and the measured velocity is slightly shifted from the velocity given by the new formula. Because the two velocities predicted by the two formulae are so close to one another for very high energy, it is very difficult to ascertain which formula corresponds to reality better.

Lund and Uggerhøj [10], two physicists from Aarhus University, Denmark, also performed a similar experiment to Bertozzi's experiment with electrons in 2009. For the two data points with very high kinetic energies and very high velocities, the velocity for a given kinetic energy is also slower than the velocity given by Einstein's formula, as predicted by the new formula (see their Figure 9). However, again we do not know which formula is closer to reality – since the two velocities given by the two formulae are so close to one another for high kinetic energy, a measured velocity with its experimental error can be a small deviation from the velocity given by either of the two formulae. Therefore, the same difficulty in ascertaining which formula is closer to reality appears both in relation to Bertozzi's data and the data by Lund and Uggerhøj. Nevertheless, there is a more promising kinetic energy range where the two velocities given by the two formulae are sufficiently different such that the difficulty described above can be overcome.

Instead of plotting normalised kinetic energy against normalised velocity squared (as in Figure 2 and Figure 4), one can plot normalised velocity against normalised kinetic energy for both formulae and work out the difference in velocity between the two formulae for a given normalised kinetic energy. Then, one can locate the kinetic energy range where the difference in velocity is a maximum. To achieve this, we need to express normalised velocity as a function of normalised kinetic energy for both formulae. For Einstein's formula, (§12a) can be rewritten as

$$\frac{v_1}{c} = \left(1 - \frac{1}{\left(\frac{K.E.}{mc^2} + 1\right)^2}\right)^{1/2} \qquad (\S 15)$$

and for the new formula, (§14) can be rewritten as

$$\frac{v_1}{c} = \left(1 + \frac{1 + \frac{m_i^2}{m^2}}{\frac{K.E.}{mc^2}\left(\frac{K.E.}{mc^2} + 2\right)}\right)^{-1/2} \qquad (\S 16)$$

where it can be seen that (§16) will reduce to (§15) if m_i^2 / m^2 is set to zero. Hence, again Einstein's formula can be seen as a special case of the new formula.

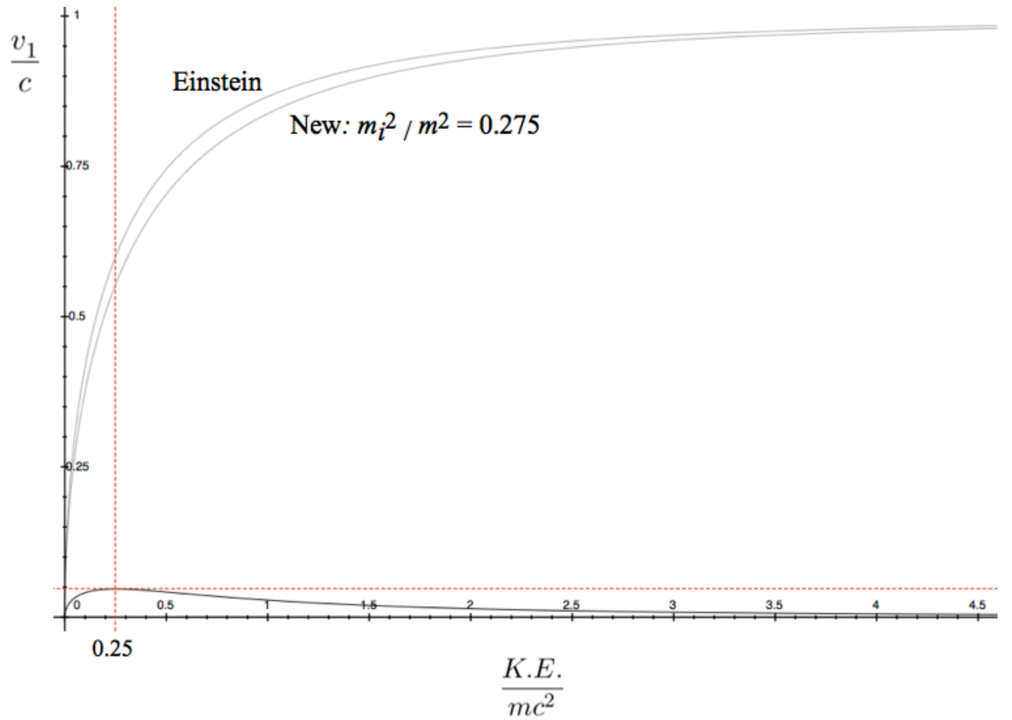

Figure 5: upper two curves – normalised velocity vs normalised kinetic energy for the two formulae; lower curve – the difference in the two normalised velocities for a given energy

Figure 5 plots normalised velocity against normalised kinetic energy for Einstein's formula and the new formula with the parameter m_i^2 / m^2 set to 0.275. To plot a sample curve for the new formula, one needs a value of the parameter m_i^2 / m^2 . The reason the value of 0.275 is chosen for this parameter is that there is one data point from Bertozzi, ($\frac{K.E.}{mc^2}$=2.945, $\frac{v_1}{c}$= 0.960), and one data point from Lund and Uggerhøj, ($\frac{K.E.}{mc^2}$=2.945, $\frac{v_1}{c}$= 0.959), which are almost identical and the curve for the new formula with the parameter m_i^2 / m^2 set to 0.275 passes through this data point, as can be seen from (§16). This is not an attempt to fit a curve of the new formula to the limited number of data points; it is merely an attempt to obtain a reasonable curve of the new formula to give an idea of the range of kinetic energy which will yield maximum difference in the velocities predicted by Einstein's formula and the new formula. It turns out that such a maximum velocity difference occurs around $\frac{K.E.}{mc^2}$= 0.25 with m_i^2 / m^2 set to 0.275. Varying the parameter m_i^2 / m^2 from 0.275, e.g., to 0.5 and 1.0, only moves the maximum difference slightly away from $\frac{K.E.}{mc^2}$= 0.25. Also, a larger m_i^2 / m^2 will yield a larger maximum velocity difference between the two formulae. For example, at $\frac{K.E.}{mc^2}$= 0.25, the velocity given by the new formula (with m_i^2 / m^2 set to 0.275) is 0.553c while the the velocity given by Einstein's formula is 0.6c, yielding a maximum difference of 0.047c. If m_i^2 / m^2 is set to 1.0, then the difference in velocity between the two formulae at $\frac{K.E.}{mc^2}$ = 0.25 is 0.132c. Table 1 summarises these results.

K.E. / mc^2	v_1 from Einstein's formula	m_i^2 / m^2 for new formula	v_1 from new formula	Difference in velocity
0.25	0.6c	0.1	0.582c	0.018c
0.25	0.6c	0.275	0.553c	0.047c
0.25	0.6c	0.5	0.522c	0.078c
0.25	0.6c	1.0	0.4685c	0.132c

Table 1: Velocities from Einstein's formula and the new formula, and their differences

The discussion above suggests that in terms of experimental verification or falsification of either of the two formulae, one should be measuring the velocity around $\frac{K.E.}{mc^2}$= 0.25 (this energy range is not covered by Bertozzi, Lund and Uggerhøj or any paper available to the author). Also, one needs to ascertain the required level of accuracy in measuring a velocity in order to prevent a measuring error from masking any genuine difference between the real velocity and a predicted velocity (predicted either by Einstein's formula or the new formula). A velocity is measured typically by measuring the time of flight covering a known distance. If, for example, the uncertainty in the time of flight is 1%, then the velocity will have an uncertainty of 1%. In that case, for $\frac{K.E.}{mc^2} = 0.25$ and with m_i^2 / m^2 set to 0.275 for the new formula *as an example*, the velocity from both formulae is around 0.6c ; *if* either of the two formulae corresponds to reality, the uncertainty in the measured velocity will be about 0.006c. One will expect the standard deviation of the measured velocities to be 0.006c in a Gaussian distribution. In the case where the new formula indeed corresponds to reality, there will be a difference of around 0.047c between the real velocity and the velocity predicted by Einstein's formula. And this difference of 0.047c is about 8 times of the standard deviation of the measured velocities (0.006c). Hence, the error in measuring the velocity will not mask the difference between the real velocity and the velocity predicted by Einstein's formula, *if* indeed there is such a difference (corresponding to $m_i^2 / m^2 = 0.275$) identified in the experiment. For m_i^2 / m^2 greater than 0.275, the difference in velocity will be even more evident but again this is assuming the new formula with that greater value of m_i^2 / m^2 is closer to reality than Einstein's formula. Whether this is the case will have to be tested by experiments as suggested above.

In any case, a curve will have to be fitted to the experimental data covering the kinetic energy range around $\frac{K.E.}{mc^2} = 0.25$ and the broader energy range beyond this. The criteria for fitting the best possible curve to the experimental data will also need to be set, e.g., chi squared minimisation or least squares. Then an optimal value of the parameter m_i^2 / m^2 will need to be chosen to satisfy the best fit criteria. If the optimal value of the parameter m_i^2 / m^2 turns out to be zero, then the curve for Einstein's formula will be the best fit curve. However, if the optimal parameter value is non-zero in satisfying the best fit criteria, then the curve for the new formula with that optimal parameter value is a better fit than the curve for Einstein's formula. In the case where the best fit curve has a very small value for the parameter m_i^2 / m^2, e.g., 0.1 (see Table 1), the curve is almost identical to Einstein's curve with a much smaller discernible difference in velocity between the two curves, 0.018c. For such a small difference in velocity not to be masked by error in measuring the velocity, an accuracy of 0.6% in the velocity uncertainty will allow 5 standard deviations of the velocity measurements to span $0.018c$, and an accuracy of 0.4% in the velocity uncertainty will allow 8 standard deviations of the velocity measurements to span $0.018c$.[7] These high accuracy levels in measuring the velocity will be sufficient to identify the discernible difference in velocity between the Einstein's formula and the new formula *if* indeed the appropriate value for m_i^2 / m^2 is 0.1. However, since we do not know the appropriate value for m_i^2 / m^2 in advance, perhaps one should begin with the highest possible level of accuracy in measuring the velocity and see if the best fit curve for the new formula with the optimal value of m_i^2 / m^2 is discernibly different from the curve for Einstein's formula, especially at the energy range around $\frac{K.E.}{mc^2} = 0.25$. The discussion so far has not included the possibility of systematic bias in the measurement of velocity. Systematic bias should not be dismissed and should be identified and minimised as much as possible to give a clearer picture of reality.

Finally, existing datasets which give the relationship between energy (total or kinetic) and translational momentum will not serve the purpose of distinguishing the two formulae for the following reason. In terms of the relationship between translational momentum and energy, (§1) (for Einstein's formula) and (§7) (for the new formula) give the same form of relationship. What makes (§7) distinctive from (§1) is their different definitions of the translational momentum and these different definitions are expressed in terms of speed in different ways in (§2) and (§5). Hence, speed (not momentum) and kinetic energy are required to unravel the distinction between Einstein's formula (§1) and the new formula (§7) and so make possible their comparison.

5.0 Discussion

Before more experimental data are made available, one cannot come to a conclusion about the validity of the new formula. However, one can consider the possible implications *if* indeed further experimental data confirm that the new formula does perform better than Einstein's formula. Such a prospective consideration about the implications is not premature

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[7] These high accuracy levels in measuring the velocity probably need a long flight path which will reduce the percentage error in the time of flight and thus reduce the percentage error in the velocity.

here because it gives us the opportunity to investigate theoretically what these implications would be.

We know from nuclear fission and nuclear fusion that there is a loss of rest mass and a corresponding increase in energy. One of the baffling questions in special relativity is about the mechanism for converting mass into energy (and vice versa) if indeed such conversion explicitly happens. Or are there other ways for explaining the loss of rest mass and the increase in energy? Bondi and Spurgin [11] pointed out that mass and energy have different dimensions and therefore suggested that there is no real conversion between mass and energy. Their explanation of mass and energy can be described in the following way. Energy has its mass-equivalent. A system's total rest mass (e.g., that of a nucleus) consists of the sum of the constituents' rest masses (e.g., those of nucleons) *and* the mass-equivalent of the system's energy. While each of the constituents' rest masses is conserved (and hence their sum is conserved), the mass-equivalent of the system's energy may vary due to exchange of energy with another system (which satisfies the requirement for the conservation of energy between the two systems). Because the contribution to the system's total rest mass from the system's energy can vary due to exchange of energy with another system, the total rest mass of the system can vary. In this case, the rest masses of a system's constituents are always conserved and such masses are *not* mysteriously converted into energy. The energy of the system has always existed as energy which can be exchanged with another system or carried away by photons, in which case the energy's contribution of its mass-equivalent to the system's total rest mass varies, rendering the total rest mass to vary. Despite the clear separation between the constituents' rest masses *and* the mass-equivalent of the system's energy, one wonders if their assertion that the constituents' rest masses are not converted into energy is correct. For example, the explanation by Bondi and Spurgin does not address the annihilation of a particle and an anti-particle whereby these particles appear to vanish and the particles' masses appear to be completely converted into energy. Also, they do not give an account of how the energy of a system can have its mass-equivalent and thereby contribute to the total rest mass of the system. Before these questions are addressed by the theory proposed by this paper, the explanation by Rindler [12] needs to be described.

Contrary to Bondi and Spurgin, Rindler asserted that matter (or the constituents of a system) and its mass can disappear and a corresponding amount of energy appears, e.g., in the annihilation of particle and anti-particle (see p. 75 of [12]). Hence, the energy produced did not exist in the form of energy but in the form of matter which had mass. But again there is the question of the details of the mechanism by which matter can suddenly disappear with the concomitant emergence of energy.[8]

5.1 How is Energy Related to Mass?

The question of how the energy of a particle is related to its mass is now addressed by the theory put forward in this paper. We deal with this question by considering the case of a particle at rest translationally. This paper proposes that a particle's inherent mass is invariant and such inherent mass is not converted into energy. When the translational velocity of a particle is zero, i.e, it is at rest translationally, it is still in a localised surfing motion on the *S* surface and the surfing motion generates its surfing momentum p_s which depends on the inherent mass and the surfing speed; see (§6). The surfing motion of the particle via its surfing momentum with its inherent mass generates the 'rest' mass of the particle, $m = p_s/c$; see (§8). And according to (§7), the surfing energy associated with the surfing momentum is $p_s c = mc^2$ which is conventionally called the rest energy. In normal circumstances, the rest mass is conserved, implying that the surfing momentum p_s and surfing energy (or rest energy) of a particle are also conserved. However, in exceptional circumstances, the surfing energy of the particle on the *S* surface can be converted into other forms of energy (e.g., particle translational kinetic energy or photon energy) when a mechanism enabling such conversion of energy takes place, e.g., in nuclear fusion or fission (which will lead to a change in the surfing momentum and its associated rest mass; see below). In this sense, the surfing energy of the particle is a kind of internal energy or potential energy which can be converted into other forms of energy. It can also be considered as a kind of 'internal kinetic energy' (kinetic because of the surfing motion) which is distinct from the external translational kinetic energy which arises from the motion in the direction perpendicular to the surfing momentum. In this view, there is no conversion of matter into energy (in agreement with Bondi and Spurgin) and there is only conversion of energy from one form to another, but, as hinted above when referencing (§8) and (§7) and as will be seen in the following, this view can give the details of the mechanism whereby the surfing energy

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[8] For a discussion of the positions of Bondi and Spurgin, Rindler and others, see [14].

$p_s c$ (or the internal kinetic energy) is related to the rest mass (*m*) of the particle (which Bondi and Spurgin, and other authors, did not address).

Physically speaking, given that in (§9)

$$p_s = \frac{m_i v_{so}}{\sqrt{1 - \frac{v_{so}^2}{c^2}}}$$

where v_{so} is the surfing velocity when the translational velocity is zero, the surfing momentum of the particle (with its inherent mass m_i) is 'whipped up' by the surfing velocity. And the rest mass of the particle is given by the 'whipped up' surfing momentum p_s divided by c according to (§8), and in that sense the rest mass is 'whipped up' by the surfing velocity.[9] Also, since the surfing energy of the particle with zero translational motion (rest energy) is given by the 'whipped up' surfing momentum p_s multiplied by c according to (§7), the ratio of the surfing energy to the rest mass is c^2 , which gives the following relationship between surfing energy and rest mass:

$$\frac{\text{Surfing Energy}}{\text{Rest Mass}} = \frac{p_s c}{p_s / c} = c^2 \implies \text{Surfing Energy} = E = E_N = mc^2 \quad (\S 17)$$

where the common factor of the surfing momentum has been cancelled and therefore does not appear in the final equation. The derived relationship between surfing energy and rest mass in (§17) replicates the well known relationship between rest energy and rest mass. This again suggests or confirms that what has been called rest energy by physicists has its origin in the surfing energy, and the two are synonymous. While rest energy is difficult to comprehend as it cannot be visualised, surfing energy can be visualised and gives us a physical meaning. Note that the derivation of the famous formula for energy here does not make reference to photons emitted from an object, as Einstein did [13], and in that sense is a more general or more fundamental derivation, independent of the theory of electromagnetism. Some physicists 'search for "purely dynamical" derivations, i.e.,

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[9] Whether this mechanism of the surfing momentum 'whipping up' the rest mass of a particle is applicable to the question of the origin of the proton mass remains to be seen.

derivations that invoke only mechanical concepts such as energy and momentum, and the conservation principles that govern them' [14].[10] In that sense, the derivation proposed in this paper is a purely dynamical derivation since it is purely based on the definitions of energy (§3), momentum in general (§4) and surfing momentum (§6) of a particle with inherent mass. It also provides an explicit visual mechanism whereby mass is related to energy via the surfing momentum of the particle whipped up by the surfing velocity; such an explicit visual mechanism involving the surfing velocity has not been presented by Einstein or other physicists.

It can be seen in the expression for mass

$$m = \frac{p_s}{c} = \frac{m_i \frac{v_{so}}{c}}{\sqrt{1 - \frac{v_{so}^2}{c^2}}} = m_i \gamma_o \frac{v_{so}}{c} \ , \ \gamma_o = \frac{1}{\sqrt{1 - \frac{v_{so}^2}{c^2}}} \qquad (§18)$$

that as the surfing velocity v_{so} increases, m increases via the increased Lorentz factor for zero translational velocity, γ_o, and the factor v_{so}. Similarly,

$$\text{Surfing Energy} = E = E_N = mc^2 = p_s c = \frac{m_i v_{so} c}{\sqrt{1 - \frac{v_{so}^2}{c^2}}} = m_i \gamma_o v_{so} c$$

where it can be seen that as the surfing velocity v_{so} increases, the rest energy also increases via the increased Lorentz factor and the factor v_{so}. In sum, a larger surfing velocity v_{so} will whip up a larger surfing momentum and therefore a larger rest mass with a larger surfing energy (rest energy). The rest mass of a particle is a useful measure of its inertia (understood as the resistance to acceleration) but it is so by virtue of its relationship with the surfing velocity and the inherent mass; see (§18). The rest mass of a particle, being the ratio of the internal surfing energy to c squared, is also a useful measure of the particle's internal energy, as Einstein's echoed in [13] in relation to a body, 'The mass of a body is a measure of its energy-content.' While he gave no further detail about how that energy resides in the body (or its constituent particles), in this paper the internal energy of a particle is explicitly

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10 See [14] for different derivations.

identified as the internal surfing energy of the particle; similarly the internal energy of a body is identified as the sum of the internal surfing energies of its constituent particles.

It can be seen that rest energy and rest mass are only related to one another via the surfing momentum, p_s , i.e., they are not intrinsically related to one another without reference to the surfing momentum. In that sense, the surfing momentum is the primary quantity through which rest energy and rest mass are related (and from which the surfing speed can be inferred; see §9). And if somehow the surfing momentum and its associated surfing energy (i.e., rest energy) are changed, these changes will be reflected in the change in the rest mass, as will be seen in the next section.

5.2 Changes in Surfing Momentum, Energy and Rest Mass

In normal circumstances, the surfing momentum (as a magnitude, i.e., as a scalar and not a vector quantity), the rest mass and the surfing energy of a particle are three scalar quantities which are all conserved and constant, regardless of the extent of the translational motion (for varying translational velocity, the surfing speed will adjust to maintain constant surfing momentum and is therefore not conserved; see §6 and §13). The constancy of these three scalar quantities has been crucial in our understanding of the generation of constant rest mass. However, in unusual circumstances as indicated above, it is conceivable that some of the surfing energy of a particle (i.e., its internal kinetic energy) is released and manifests itself in other forms of energy, e.g., photon energy or particle translational kinetic energy. In such unusual circumstances, the change in surfing energy $p_s c$ is concomitant with the change in surfing momentum and the consequent change in rest mass $m = p_s/c$. Since the surfing energy $p_s c = mc^2$ is such a large quantity, a small percentage change of this energy amounts to a very large change in this energy in absolute terms. Even a small percentage decrease of this energy amounts to a huge loss of this internal kinetic energy in absolute terms and the huge energy lost re-emerges itself in a different form (or different forms) of energy. In this process of energy conservation and energy transformation, the rest mass as a secondary quantity, derived from the primary quantity of surfing momentum, changes but the inherent mass, which is a fundamental quantity, remains unchanged (this means the parameter for a particle, m_i^2 / m^2 , is normally constant but may change in unusual circumstances because *m* may change). This may be what happens in nuclear fission or fusion where a comparatively small decrease of rest mass is concomitant with a loss of surfing momentum and a loss of surfing internal kinetic energy (according to this paper's interpretation) which are accompanied by a huge increase in external energies, i.e., energies which are of forms different to the internal kinetic surfing energy, e.g., photon energy or particle translational kinetic energy. Further research is necessary to further elucidate details of such energy transformation.

In Einstein's thought experiment [13], he employed two inertial frames and invoked two photons emitted from a body to derive the change in rest mass of the body as a result of its loss of energy (which has emerged as photon energy). In this paper, the mass-energy relationship has been obtained without the reference to such inertial frames or photons being emitted. Nevertheless, the mass-energy relationship obtained from the fundamental derivation in this paper can be applied to his thought experiment to give a satisfactory explanation of it – an equal amount of surfing internal energy is lost by two particles which emit the energies in the form of photon energy in opposite directions and the loss of surfing internal energy is simultaneously accompanied by a loss of surfing momentum and therefore a loss of rest mass for these two particles. In similar ways, the mass-energy relationship obtained from the fundamental derivation in this paper can be used to explain other thought experiments involving that relationship; see [14] for other thought experiments.

Rindler [12] asserted that matter and its mass can disappear and a corresponding amount of energy appears. But so far the alternative theoretical analysis from the paper's theory shows that some energy can appears from some internal source without any matter disappearing. Therefore, the particle's ontological existence needs not be diluted or threatened and the particle's inherent mass can remain intact. Furthermore, a counter-example to Rindler's scenario of matter disappearing is this: a particle lost a fraction of its rest energy, mc^2 , but this fractional loss of rest energy cannot be accounted for by a fractional disappearance of the corresponding discrete particle with rest mass *m* (which is nonsensical). But this fractional loss of rest energy can be accounted for in this paper by a fractional loss of the particle's surfing momentum which is a continuous variable. Even though this counter-example raises difficulty for conceiving the disappearance of a fraction of a particle and the fractional decrease in rest energy, the case of annihilation of particle and anti-particle, with the complete loss of their rest energies, raises the question of loss of matter more acutely. This serious question is addressed in the next section.

5.3 Annihilation of Particle and Anti-particle

We now deal with the annihilation of particle and anti-particle where these matters and their masses *seem* to disappear and a corresponding amount of energy appears (cf. Rindler). The crucial questions here are whether these matters actually disappear and whether there is an alternative explanation of the the observed phenomenon of 'annihilation'.

If we take the above picture of the decrease in surfing energy to its extreme case, i.e., to zero surfing energy, then the particle has zero surfing momentum and zero rest mass. The zero surfing momentum also means zero angular momentum since the spinning velocity, $\underset{\sim}{v}_2$, is also zero.[11] It is conceivable such a total loss of surfing energy, surfing momentum, rest mass and angular momentum takes place when a particle and anti-particle come together. Before coming together, the particle and anti-particle have the same inherent mass, the same rest mass, the same surfing energy, the same surfing momentum, the same magnitude of angular momentum but of opposite signs (if the angular momentum is non-zero), and the same charge but of opposite signs (if the charge is non-zero). As they come together, the surfing energy is completely given up by the particle and anti-particle such that each of their surfing energy is zero, with the concomitant zero surfing motion, zero surfing momentum, zero rest mass and zero angular momentum. The previous surfing internal kinetic energy of the particle or the anti-particle was $p_s c = mc^2$. After they have come together, these two surfing internal kinetic energies, which have been lost, re-emerge themselves in two photon energies. What happens to the ontology of the particle and anti-particle after coming together? Contrary to the conventional understanding of annihilation, this paper suggests that the particle and anti-particle have not mysteriously disappeared ontologically but they still exist as a joint entity, albeit with zero surfing momentum, zero surfing energy, zero rest mass, zero angular momentum and zero charge (i.e., if they had opposite charge before joining together, after joining their summed charge is zero). This makes the joint entity

[11] Conventionally photons are understood as having zero mass but this will imply that the spin of a photon is zero. Since a photon has non-zero spin, this paper suggests that a photon has extremely small but non-zero mass. See Section 8.5 of [8]. The mass of photons will be dealt with further in a future paper.

almost non-existent in terms of (i) energy, (ii) surfing momentum, (iii) rest mass, (iv) angular momentum and (v) charge. Nevertheless, even though the latter four quantities are genuinely zero, and even though the surfing energy is also zero, the translational kinetic energy of such a 'massless' joint entity can be non-zero if its translational speed and therefore its translational momentum is non-zero (see §5) – the non-zero translational momentum gives rise to this non-zero energy:

$$E_N = p_{1N}c$$

according to (§7) where the inherent mass of the joint entity, appearing in p_{1N}, is understood to be twice m_i. In this case, all the energy resides in the translational motion since there is no surfing motion. Note that if we apply Einstein's formula to this joint entity, since the rest mass is zero, the translational momentum is zero according to (§2); the kinetic energy and the total energy, *E*, will also be zero according to (§1).

But does the above notion of non-zero energy for a joint 'massless', spin-less and charge-less entity formed from 'annihilation' (or it may be better to use the term 'coming together') have any experimental or observational evidence to support it? It is possible to consider this notion in relation to the problem of the elusive dark matter and the energy thereof, i.e., it is possible to use the above theoretical notion to interpret or explain dark matter which thus serves as a possible observational evidence in support of the theoretical notion. This of course will require more detailed research but a preliminary sketch of the possible correspondence between the theoretical notion and the observed phenomenon of dark matter can be made here.

5.4 Dark Matter and Its Energy

We begin with the hypothesis that 'massless' entities, joint from particles and anti-particles, exist ontologically as described above. These 'massless' entities are massless only in the sense of zero rest mass but their inherent masses are non-zero and their energies can also be non-zero. There could be many such joint massless entities spread out in the vast space in a galaxy with non-zero translational kinetic energies. These joint entities are undetectable in terms of their rest mass, spin and charge but their presence is manifested through their translational kinetic energies which add to the total energy of the galaxy. And when such a hidden joint entity receives sufficient energy, i.e., greater than twice the rest energy of the particle (or anti-particle), the particle and anti-particle in the joint entity will be separated, or 'created', each with its own surfing momentum (p_s) and therefore its own rest mass ($m = p_s/c$) and its own surfing energy ($p_s c = mc^2$), its own spin (because of the non-zero surfing motion) but of opposite signs in order to conserve the total angular momentum,[12] and its own charge (if any) of opposite signs. Effectively, this particle and anti-particle 'creation' mechanism is the reverse of the 'annihilation' mechanism. This manner of envisaging particle 'annihilation' and 'creation' has the advantage that it does not involve the mysterious disappearance of matter at 'annihilation' and the mysterious appearance of matter at 'creation'. These matters, or more precisely these particles and anti-particles, are always there but they either exist individually with non-zero rest mass or exist in a joint particle and anti-particle form with zero rest mass. In either case, they could contribute energy to the total energy of the galaxy to hold it together gravitationally. In this preliminary sketch relating ontologically existent 'massless' joint particle pairs to dark matter, it seems that the ideas of surfing momentum, rest mass, and surfing energy can be at least compatible with dark matter which is used as a term for the observed energy deficit of a galaxy. More research is required to investigate this compatibility further. Also, further research is necessary to relate the proposal put forward here, concerning 'annihilation' and 'creation', with quantum field theory and the Standard Model where annihilation and creation have different meanings.

5.5 Mass and Energy – Equivalence and Conversion?

We now return to the idea of the conversion of mass into energy (or vice versa) as proposed by Rindler [12]. The case of partial loss of surfing momentum, partial loss of rest mass and partial loss of surfing internal kinetic energy was considered in Section 5.2. The case of total loss of surfing momentum, rest mass and surfing internal kinetic energy –'annihilation'– has been considered in Section 5.3. In both cases, particles do not need to disappear (or in reverse re-appear) mysteriously in the process of energy production (or energy consumption)

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[12] If the particle has zero spin, i.e., $\underset{\sim}{v}_2$ is zero, it can still have surfing motion and thus non-zero surfing momentum because $\underset{\sim}{v}_s \equiv \underset{\sim}{v}_2 + \underset{\sim}{v}_3$.

– there needs not be conversion of ontological matter into energy (or vice versa) as Rindler suggested. *Because rest mass has long been associated with existing matter, a reduction in rest mass has been interpreted as the disappearance of some matter [12]. However, this paper suggests that rest mass is a secondary derived quantity whose reduction does not entail the disappearance of some matter, but the reduction of this derived quantity of rest mass is concomitant with the reduction of internal surfing energy and the reduction of surfing momentum of continuously existing matter*. This paper therefore agrees with Bondi and Spurgin [11] that there is no mysterious conversion between ontological matter and energy but it has gone beyond them to provide a possible explanation for the clear ontological distinction between matter and energy in the processes responsible for (i) the partial loss of rest mass and (ii) the total loss of rest mass ('annihilation'). It may be more appropriate to conceive matter as *continuously existing* particles with their inherent masses which cannot be diluted or annihilated, and conceive the energy of a particle as carried by the particle in its internal surfing motion or its external translational motion, or both, as seen in $E_N^2 = p^2c^2 = p_{1N}^2c^2 + p_s^2c^2$. A particle can gain or lose energy through the change in its translational momentum (more common) or the change in its surfing momentum (in unusual circumstances). Before 'annihilation', i.e., before a particle and an anti-particle come together, in normal circumstances each of their total energy is dominated by the surfing (internal kinetic) energy. If they come together in 'annihilation' to form a joint entity, this 'massless' entity, which is nevertheless ontologically existent, only has translational (external kinetic) energy, i.e., zero surfing (internal kinetic) energy. This paper suggests that the apparent conversion of mass or matter into energy when they come together could actually be a conversion of their surfing energies into another form of energy (photon energy) while the ontological nature of the particle and the anti-particle with their unchanging inherent mass remains intact, i.e., they continue to exist with their inherent mass. As Bondi and Spurgin [11] pointed out, because there is an incommensurability between the dimension of mass and the dimension of energy, the concept of conversion from one to the other is problematic. There is certainly commensurability between kinetic energy and potential energy, or between translational external kinetic energy and surfing internal kinetic energy since they have the same dimension, i.e., energy. Because of such commensurability in dimension, one can speak of their *equivalence*. However, since we do not have this kind of commensurability in dimension between energy and mass, energy and mass are different quantities of different categories and it is conceptually difficult to speak of their *equivalence* even though they are related to one another (i.e., through the surfing momentum), just as it is difficult to speak of the *equivalence* between mass and volume even though they are related to one another via mass density.[13] Therefore, *strictly speaking it may be more appropriate to assert that mass and energy are related to one another rather than they are equivalent to one another*. The interpretation of energy and rest mass put forward in this paper notes the evident incommensurability of the two different dimensions for mass and energy and avoids speaking of the conversion of mass into energy (or vice versa) and their equivalence which are conceptually problematic. Nevertheless, the interpretation is able to account for the change in rest mass which is accompanied by the release or consumption of energy from one form to another.

6.0 Conclusion

Hestenes [1,5], Lepadatu [6] and Rockenbauer [7] associated the rest energy and rest mass of a particle with the spinning motion of the particle. Their approaches require the particle to move at the speed of light. This paper echoes their approaches but takes quite a different route where (i) the particle is not required to move at the speed of light, and (ii) the rest energy and rest mass of a particle are associated with the surfing velocity of the particle on the phase (*S*) surface, $\underset{\sim}{v}_s \equiv \underset{\sim}{v}_2 + \underset{\sim}{v}_3$, which in turn feeds into the all important surfing momentum, p_s. $\underset{\sim}{v}_2$ can be called a spin velocity component but there is also the other component in the surfing velocity which they did not consider, $\underset{\sim}{v}_3$, which is perpendicular to $\underset{\sim}{v}_2$. Furthermore, this paper has worked out systematically the details of how the surfing momentum is related to the rest/effective mass ($m = p_s/c$) and the surfing energy ($E_N = p_s c = mc^2$ for zero translational velocity) by invoking the notion of inherent mass, m_i, which is absent in their models. This yields $\text{Surfing Energy} = mc^2$. Comparing this formula with the famous formula for rest energy, $E = mc^2$, shows that the surfing energy can

[13] Bondi and Spurgin [11] gives the analogy of the relationship between mass and volume which are related via density but they have different dimensions and they are different quantities which cannot be converted to one another.

be interpreted as the origin of what has been called rest energy by physicists. While rest energy is difficult to visualise and comprehend, surfing energy can be intuitively visualsed as a kind of internal kinetic energy to give us a comprehensible physical meaning. For non-zero translational velocity, the surfing internal kinetic energy is distinct from the translational external kinetic energy, and by definition (§12b) the sum of these two kinetic energies gives the total energy of the particle. This total energy of the particle can be simply written as

$$E_N \equiv pc$$

according to (§3) where p is the total momentum of the particle (§4) which incorporates the surfing momentum p_s and the translational momentum p_{1N} whose directions are perpendicular to each other. It is seen from this definition of energy that the total energy of a particle is simply proportional to its total momentum (whose definition involves the inherent mass, m_i, rather than the rest mass; see §4). This simple proportional relationship between momentum and energy may also be considered as intuitive, e.g., doubling the momentum will double the energy. In special relativity and in Newtonian mechanics, the relationship between energy and translational momentum (or velocity) is considerably more complicated than the one proposed in this paper:

$$E = mc^2 \left(1 + \left(\frac{p_{1E}}{mc}\right)^2\right)^{1/2} \text{—— Special Relativity}$$

$$E = mc^2 + \frac{1}{2}mv_1^2 \text{—— Newtonian Mechanics .}$$

It ought to be pointed out that the above derivation of the formula for surfing energy or rest energy, $E_N = p_s c = mc^2$, does not make use of the idea of two photons emitted by a body, as Einstein did. In that sense, the formula derived here is independent of the theory of electromagnetism.[14] It is purely a derived consequence of the simple definition of energy (§3) and the definition of momentum (§4). In that sense, this derivation is more general or more fundamental.

In special relativity and in Newtonian mechanics, mass or rest mass is treated as a fundamental or primary quantity which is not defined or derived in terms of other fundamental or primary quantities. As a fundamental quantity, the rest mass of a body has been understood to correspond to how much matter there is in the body, or more precisely how many particles of various types there are in the body. However, the theory in this paper suggests that mass (i.e., rest mass) is a derived secondary quantity defined in terms of the surfing momentum and c, $m = p_s/c$, such that rest mass can decrease while matter is conserved. In normal circumstances, the surfing momentum and hence the rest mass are also conserved, so is the surfing energy. However, in unusual circumstances, e.g., those in nuclear fusion or fission, it is conceivable that the surfing momentum and surfing energy are reduced, leading to a reduction in the rest mass.[15] The surfing (internal kinetic) energy which has been lost manifests itself in other energy forms, e.g., photon energy or translational kinetic energy carried by particles in a process of violent energy exchange. In this process of energy transformation, there is no loss of matter or particle and the fundamental or primary inherent mass of the particle remains intact, i.e., there is no reduction of the particle's inherent mass. Only the rest mass, which is a secondary derived quantity, is reduced. In the extreme case where the surfing (internal kinetic) energy is completely exhausted and transformed into other forms of energy, the secondary quantity of rest mass is reduced to zero but the fundamental inherent mass of the particle still remains intact, and there is no mysterious disappearing of the particle whose inherent mass is annihilated. The paper suggests that this extreme case of total loss of surfing energy and rest mass could happen

[14] However, one can still argue that the constant, *c*, in the definitions of energy and momentum adopted here comes from the theory of electromagnetism. Nevertheless, one can respond that *c* is a universal constant which appears in electromagnetism and in the definitions of energy and momentum for particles. In that sense, these definitions and the consequent formula for energy are independent of the theory of electromagnetism.

[15] At the end of his paper, Rockenbauer hypothesised, 'In the fusion or fission processes of atomic nuclei, the vast energy of escaping irradiation is supplied by the partial loss of the spinning kinetic energy of nucleons.' This paper almost agrees with his hypothesis but will replace the term 'spinning kinetic energy' with 'surfing kinetic energy'. This could have implications for our understanding of strong nuclear force.

when a particle and its anti-particle come together, resulting in a joint entity with zero surfing energy, zero surfing momentum, zero rest mass, zero angular momentum and zero charge while its translational kinetic energy can be non-zero. The particle and the anti-particle are not annihilated but continue their ontological existence in a joint state, albeit in a less detectable form. (When such a joint entity receives sufficient energy, the previous process of coming together can be reversed, i.e., the joint entity is divided giving rise to a particle and an anti-particle, both with non-zero equal rest mass.) The existence of the less detectable joint entities without rest mass but with translational kinetic energy could account for the missing energy – dark matter – necessary for holding a galaxy together gravitationally. More research in this area is necessary.

Bondi and Spurgin [11] pointed out the incommensurability between the dimension of mass and the dimension of energy. In light of this, the concept of conversion from mass to energy (as suggested by Rindler [12]), or vice versa, is problematic. Quantities with the same dimension can be converted from one form to another, e.g., potential or internal energy can be converted to kinetic energy. For the same reason, strictly speaking it is also problematic to speak of the equivalence between mass and energy of different dimensions. It is more appropriate to assert that rest mass and energy are related to one another (via the surfing momentum) rather than they are equivalent to one another.

The theory of mass and energy proposed in this paper is based on the the relativistic model for quantum particles which heavily utilises the notion of surfing motion on the S surface. That model was briefly explained in Section 2 of this paper but it has been shown in [8] that the model is consistent with or give credible explanations for a significant number of *observed phenomena* in quantum mechanics: (i) non-determinacy, (ii) de Broglie relation between momentum and wavelength, (ii) Planck-Einstein relation between energy and frequency, (iii) Born's rule, (iv) spin, (v) ontological particle nature of *both* particles and photons, (vi) the observed wavy functional behaviour of *both* particles and photons, (vii) the interference pattern observed in the two-slit experiment, (viii) measurement which destroys the interference pattern of the two-slit experiment and (ix) tunnelling (for these, see Section 10.2 on Conclusion in [8]). This paper focuses on how that model can be used to unravel (x) the *observed* relationship between rest mass and energy.

The philosophy of science teaches us that no theory can be conclusively proved to be correct since in principle every theory is falsifiable and could be improved by a better theory.

One can only speak of the credibility of a theory or model and its proximity to reality. The credibility of a theory or model should be gauged according to its consistency with observation, no matter how elegant a theory or model is. The relativistic model for quantum particles proposed in [8], on which this paper is based, and the theory of mass and energy proposed in this paper, can have some claim to elegance, e.g., the prescription of a set of three orthogonal velocities including the two crucial surfing velocity components on the *S* surface, and the simple expression for the new definition of energy, $E_N \equiv pc$.

Concerning the claim to consistency, even though [8] and this paper claim to be consistent with the above ten observed phenomena, it has been suggested in Section 4 above that further experiments ought to be carried out to verify or falsify the theory's proposed consistency with the relationship between translational kinetic energy and velocity as observed in experiments. Section 4 points out that experimental data, especially with normalised kinetic energy around 0.25, could decide whether the new equation (§16), derived from the theory, relating velocity with kinetic energy, is better or worse than the conventional equation from Einstein (§15). Because the two equations can be very similar to one another due to a possible small value for the parameter, $m_i^2 \,/\, m^2$, it has been suggested that a very high accuracy for measuring the velocity of a particle will be necessary to distinguish the two equations. If experiments show that the new equation performs better than the conventional equation from Einstein, then these experimental results will give some credibility to the theory put forward in this paper and add further credibility to the relativistic model for quantum particles put forward in [8]. In that case, the further research and implications as suggested in this paper should be pursued.

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