Mass in a Fokker-Type Theory

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ABSTRACT

Despite the efforts of Wheeler, Feynman, Davies, Hoyle, Narlikar and others, historical attempts to reconcile direct particle interaction of Schwarzschild, Tetrode, and Fokker with Maxwell field theory have failed, forcing the conclusion that time-symmetric EM fields cannot be entirely expunged from direct particle interaction. Though electromagnetic time-symmetric fields have hitherto been deemed an ‘undesirable’ prediction of direct particle interaction, it turns out that observational facts peculiar to QM may instead be consistent with their presence, in which case direct particle interaction may be a viable theory after all.

The alleged role of time-symmetric EM fields in the emergence of quantum theory from a ‘classical’ mostly time-symmetric background is discussed elsewhere. This work instead examines a particular consequence of the self-consistency that must be demanded of such fields presuming they exist. The motion of sources in the presence of such fields is shown to constrain their mass, which appears as an eigenvalue in controlling self-consistent modes over cosmological scales. Calculation of the eigenvalue under the presumption of a uniform distribution of matter yields a relationship between the electron mass and the Hubble radius consistent with one of the Dirac Large Number Hypotheses.

More generally, it is shown that direct particle interaction predicts a relationship between the mass spectrum of elementary particles, and the distribution of matter over Cosmological scales.

Keywords: Direct particle interaction, time-symmetry, electron mass, Fokker, Cosmology

1 INTRODUCTION

Direct inter-particle interaction (henceforth 'direct action') is a contender for the replacement of field-theoretical electrodynamics. The classical implementation discussed here was first investigated by Schwarzschild, Tetrode, and Fokker, the latter having since become associated with the corresponding action, one particular form of which is [1]

$$I_{Fokker} = -\int \mathrm{d}^4 x \int \mathrm{d}^4 x' G_{sym}\left(x - x'\right) j\left(x\right) \circ j\left(x'\right) \tag{1}$$

$j\left(x\right)$ is the total classical current, which can be written

$$j\left(x\right) = \sum_k e_k \int \mathrm{d}\kappa v_k\left(\kappa\right) \delta^4\left(x - q_k\left(\kappa\right)\right); \quad v_k\left(\kappa\right) \equiv \mathrm{d} q_k\left(\kappa\right) / \mathrm{d}\kappa \tag{2}$$

The dynamics are determined by extremization of the total action

$$I = I_{mech} + I_{Fokker} \tag{3}$$

by variation of the $q_k\left(\lambda\right)$, where, traditionally,

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$$I_{mech} = -\sum_l m_l \int \mathrm{d}\lambda \sqrt{v_l^2\left(\lambda\right)} \tag{4}$$

Explicitly therefore

$$I = -\sum_l m_l \int \mathrm{d}\lambda \sqrt{v_l^2\left(\lambda\right)} - \sum_{k,l} e_k e_l \int \mathrm{d}\kappa \int \mathrm{d}\lambda G_{sym}\left(q_k\left(\kappa\right) - q_l\left(\lambda\right)\right) v_k\left(\kappa\right) \circ v_l\left(\lambda\right) \tag{5}$$

Electromagnetic self-action can be excluded by excluding the 'diagonal terms' $k = l$ in the double-sum.

$G_{sym}\left(x - x'\right)$ is the scalar time-symmetric Green's function satisfying $\partial^2 G_{sym}\left(x\right) = \delta^4\left(x\right)$. In the purely classical version of direct action $j\left(x\right)$ is a collection of distinguishable classical 4-currents due to point charges, the positions of which are the only dynamical variables in the action. Since the vector potential is absent from the action, direct action admits no *independent* field degrees of freedom. This restriction does not preclude the use of fields in the description of the dynamics, once the latter have been specified by extremization of the Fokker action (1) plus some form of mechanical action. Consequently Fokker differs from field theory (at least) in all cases when the potential cannot be eliminated from the latter. Necessarily these cases involve interactions between charges and radiation, the latter understood as comprising genuine field degrees of freedom of the vacuum in the traditional Maxwell theory. Generation of radiation that is eventually absorbed − no matter how distantly − in principle can be described in terms of current-current interactions, and so is excluded by this definition.

1. $u \circ v$ is the scalar product of two Lorentz vectors, and $u \circ u = u^2$. $x = \left(t, \mathbf{x}\right)$ is a 4-coordinate.

Note that the structure of the Fokker action is such that it vanishes automatically if G_{sym} is replaced with the anti-symmetric $G_{a-s}(x) = (\Theta(t) - \Theta(-t))G_{sym}(x)$. It follows that the action is unchanged (apart from an overall factor of 2) if G_{sym} is replaced with $G_{ret}(x) = 2\Theta(t)G_{sym}(x)$ or $G_{adv}(x) = 2\Theta(-t)G_{sym}(x)$. Specifically:

$$\int \mathrm{d}^4 x \int \mathrm{d}^4 x' G_{a-s}(x - x') j(x) \circ j(x') = 0 \; \forall \; j(x) \tag{6}$$

It follows that use of the symmetric Greens function is *not* in fact a distinguishing feature of direct action.

Complete accord between direct action and field theories (the classical Maxwell theory and the corresponding second-quantized theory), and hence with observation, relies on the eventual absorption of *all* retarded radiation, tying the fate of direct action to Cosmology. Equivalence of the two theories assuming all radiation is eventually absorbed was demonstrated by Wheeler and Feynman, and subsequently Davies. The reader is referred to Davies (The Physics of Time Asymmetry, University of California Press, 1974) for a detailed discussion of the relevant Cosmological and thermodynamic issues. Subsequently, particularly as a result of the development of Cosmology and the discovery of accelerating expansion, it was found that the density of charged matter on the future light cone extending to infinity is insufficient to guarantee absorption of locally sourced radiation (from our era, for example).

Those attempts to reconcile Fokker theory with field theory are predicated on the emergence of a response from distant charges to the acceleration of a local charge that appears as an 'in-coming' anti-symmetric Faraday at the local charge with a magnitude exactly equal to the acceleration component of the 'out-going' time-symmetric Faraday of that charge so that $F_{sym} + F_{a-s}$ is completely retarded. In other words, the advanced part of the out-going Faraday is entirely cancelled by the response. Here we depart from that 'tradition' and do not presume there is perfect cancellation. That is, the magnitude of the anti-symmetric response – to the extent it exists - is insufficient to cancel the advanced part of the out-going acceleration-field Faraday generated by a time-symmetric Green's function. The effect of this departure is two-fold. It removes the Wheeler and Feynman constraint of perfect absorption by distant charges, and it leaves a residual time-symmetric component of the field that must somehow be reconciled with the facts of observation. Given the failure hitherto to find the necessary absorbers, the former opens the door to rehabilitation of the Fokker theory. The focus of this paper is on the latter.

2 SELF-CONSISTENT MASS

2.1 INTRODUCTION

Post recombination, field theory maintains that the matter and EM field degrees of freedom are mostly decoupled. Due to the presence of both advanced and retarded interactions it is doubtful that in a Fokker theory however matter can ever be completely decoupled electromagnetically. Nonetheless, in practice, in the event an emergent anti-symmetric part of an interaction may appear to have its own field degrees of freedom, we must presume that the 'radiation' component of the interaction behaves *as if* decoupled from matter. But unless the field pattern that emerges from the collective behavor is entirely retarded (which it is not), there will remain significant electromagnetic coupling that does not go away, post recombination.

In this highly idealized treatment we consider the consequence of the existence of time-symmetric EM fields (i.e. ignoring retarded radiation) on free charges in Minkowski spacetime in the non-relativistic regime. We do not dwell here on the details of the motion of an individual charge in a background of self-consistent fields, but rather on the macroscopic features of the time-symmetric Fokker interaction. Only the response of the electrons (and positrons, in the event they turn out to be significant) need be considered since in total theirs is the most significant contribution to the total scattering surface (about 4 x 10^6 larger than the surface presented by protons, for example).

2.2 NON-RELATIVISTIC SELF-CONSISTENT FIELDS

Let $\mathbf{a}_i(t) = \ddot{\mathbf{x}}_i(t)$ be the ordinary acceleration of a typical electron whose mean position is $\mathbf{x}_i$, and instantaneous position is $\mathbf{x}_i + \mathbf{x}_i(t)$. We presume each such electron obeys a non-relativistic equation of motion consistent with the dipole approximation

$$m_e \mathbf{a}_i(t) = e_i \mathbf{E}(t, \mathbf{x}_i) \tag{7}$$

where $\mathbf{E}(t, \mathbf{x}_i)$ is the adjunct electric field of all other charges. We will assume that the particles are all in the far-field of each other at the frequencies of significance. In that case the electric field of other charges can be written

$$\mathbf{E}(t, \mathbf{x}_i) = -\sum_{\substack{j \\ j \neq i}} \frac{e_j}{2r_{ij}} \mathbb{U}(\mathbf{x}_{ij}) \mathbf{a}_j(t + \sigma_{ij} r_{ij}) \tag{8}$$

where $\mathbf{x}_{ij} = \mathbf{x}_i - \mathbf{x}_j$ and $r_{ij} = |\mathbf{x}_{ij}|$ is the separation between particles. $\mathbb{U}(\mathbf{x}) \equiv 1 - \hat{\mathbf{x}}\hat{\mathbf{x}}^T$ is a 3x3 projection matrix that removes the longitudinal component of acceleration. $\sigma_{ij} = \pm 1$ according to the time-order of the two particles involved in each interaction. Due to the generally great distances involved we should treat the position of the j^{th} particle on the forward and backward lightcones of a particle i at a fixed time - whose relative value is $\mathbf{x}_{ij} = \mathbf{x}_i - \mathbf{x}_j$ - as unrelated due to the effect of even a very small 'secular' drift. Thus if N particles are present there are taken to be $2N-2$ terms in the sum in (8). Using (8) in (7) gives

$$\mathbf{a}_i(t) = -\frac{1}{2m_e} \sum_{\substack{j=1 \\ j \neq i}}^{2N} \frac{e_i e_j}{r_{ij}} \mathbb{U}(\mathbf{x}_{ij}) \mathbf{a}_j(t + \sigma_{ij} r_{ij}); \quad \sigma_{ij} = \pm 1 \tag{9}$$

Eq. (9) is a self-consistency condition on the acceleration of the electrons, and can be expressed as an eigenvalue problem in which the set of $2N$ accelerations is $2N \times 3$ component vector. Calculation of the eigenvalue associated with a uniform distribution of charges in a bounded Minkowski spacetime is straightforward and given in Appendix A. Given N particles in a 3-space of radius R we recover the Dirac large number relation [2]

$$m_e = \sqrt{N} e^2 / 4\pi R \tag{10}$$

From the perspective of the Friedmann equation this particular 'coincidence' is a snapshot of a time-varying relationship between the scale factor and the mass-density. From the perspective of the dynamics of an isolated charge this result establishes a role for the time-symmetric fields in deciding the value of the electron mass. In the Fokker paradigm therefore,[3] the electron mass cannot be dominated by a fixed intrinsic mechanical inertia because its value is in large part, if not entirely, determined by the collective electromagnetic response of other charges.

Note that the derivation of (10) does not permit the inference that this mass-energy *resides* in the interactions mediated by time-symmetric fields; one can infer only that its value is adjusted so that it conforms to a constraint mediated by these fields. In fact, further investigation of the fields employed reveals that the interaction energy, excluding self-action, is entirely negative, suggesting that electromagnetic time-symmetric fields mediate gravity. The same result can be obtained more easily - bypassing calculation of the acceleration fields - via use of the virial theorem.

2.3 DYNAMIC MASS

In a more relatistic Cosmology we infer from the above that the Fokker paradigm demands a model of electron mass that adjusts with the expansion. Note that this is not inconsistent with accepted physics since the Dirac wavefunction satisfies

$$\left(\partial - i m_e a(t)\right)\psi(x) = 0 \tag{11}$$

in a conformal spacetime with scale factor $a(t)$. Here $m_e a(t)$ is effectively a *dynamic* mass.[4] In any case, we infer that classical Fokker theory requires that the fixed non-relativistic mechanical mass-action of the electron - implicit in the derivation of (10) - must be replaced with a dynamic mass.

In conformal spacetime the traditional mechanical contribution to the classical action is given as

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2. Since the calculation presumes homogeneity the 3-space boundary can be regarded as the radius of communication in an unbounded space.

3. The Fokker paradigm, that is, in which the advanced fields are not presumed to have been completely cancelled out.

4. This is another indication of the likely emergence of gravity from a Fokker theory: The scale factor $a(t)$ – which is the cosmological gravitational field given to us by GR – emerges instead from the Fokker theory with speed-of-light electrons as a Lagrange multiplier, whose value is ultimately determined by the distribution of distant matter.

$$I_{mech} = -m_e \int \mathrm{d}t a\left(q(t)\right)\sqrt{v^2(t)} \tag{12}$$

As it appears in (12) the scale factor $a\left(q(t)\right)$ traditionally comes with the presumption that it is determined by the machinery of of GR (e.g. the Friedmann equation in the case of conformal cosmology). Since there is no room in that theory for a role for time-symmetric fields the mechanical term must be discarded in favor of a mass of purely electromagnetic origin, which in this case is restricted to self-action – now of a *massless* bare charge.

Electromagnetic self-action - whether in the Maxwell or Fokker theory - predicts infinite self-energy, unless perhaps the charge moves at light speed.[5] What happens at light speed depends critically on how that limit is approached, and therefore on the precise specification of the action. Since EM is scale-invariant, if the action is such that the mass is finite at light-speed then it is zero there. A particular limiting procedure that gives zero self-energy at light speed and infinite energy at other speeds [6] will be given elsewhere. To integrate this behavior into a Fokker action that previously omitted self-action we include for each charge an additional contribution

$$I_{mech} \to -\int \mathrm{d}\lambda \mu(\lambda) v^2(\lambda) \tag{13}$$

where $\mu(\lambda)$ is an undetermined multiplier whose job is to enforce $v^2(\lambda)=0$. This action has an alternative interpretation: it is the electromagnetic self-action of a charge infinitesimally displaced from light-speed, where $\mu(\lambda)$ is a degree of freedom of the charge embodying that infinitesimal displacement. We recover the mass-dependency demanded by time-symmetric fields by writing $\mu(\lambda)=½m_e f(\lambda)$, observing that $f(\lambda)$ is a dimensionless degree of freedom determined from the dynamics arising from interactions with other charges.[7] In laboratory-time form the dynamical mechanical action is then

$$I_{mech} = -\frac{1}{2} m_e \int \mathrm{d}t f(t) v^2(t) \tag{14}$$

Other actions that achieve the same end are discussed elsewhere.

2.4 RELATIONSHIP BETWEEN FOKKER AND AN EMERGENT FIELD THEORY

Introducing the adjunct potential of all but one particular charge (denopted by label *l*):

5. Under some conditions the self-energy of a charge in superluminal motion can also be rendered finite.

6. Recall that the charge in question now has no mechanical mass.

7. $f(\lambda)$ is specific to the worldline of each charge whereas the general-relativistic conformal metric – of the FRW Cosmology for example – is effectively common to a very large number of charges. Detailed discussion of the connection between these two is outside the scope of this article.

$$A_{\bar{l}}(x) := \sum_{\substack{k\\k\neq l}} e_k \int \mathrm{d}\kappa G_{sym}\left(x - q_k(\kappa)\right) v_k(\kappa) \tag{15}$$

the action in (5) with a traditional classical mechanical component can be written as $I = \sum_l I_l$, where

$$I_l = -m_l \int \mathrm{d}\lambda \sqrt{v_l^2(\lambda)} - \int \mathrm{d}^4 x A_{\bar{l}}(x) \circ j_l(x) = -\int \mathrm{d}\lambda \left\{ m_l \sqrt{v_l^2(\lambda)} + e_l A_{\bar{l}}\left(q_l(\lambda)\right) \circ v_l(\lambda) \right\} \tag{16}$$

$A_{\bar{l}}\left(q_l(\lambda)\right)$ is the potential of all but the l^{th} charge evaluated on the worldline of the l^{th} charge. The associated Euler equations are

$$\begin{aligned} m_l \frac{\mathrm{d}}{\mathrm{d}\lambda} \frac{v_l(\lambda)}{\sqrt{v_l^2(\lambda)}} &= e_l \left(\frac{\partial}{\partial q_l(\lambda)} A_{\bar{l}}\left(q_l(\lambda)\right) \circ v_l(\lambda) - \frac{\mathrm{d}}{\mathrm{d}\lambda} A_{\bar{l}}\left(q_l(\lambda)\right) \right) \\ &= e_l \left[\partial\left(A_{\bar{l}}(x) \circ v_l(\lambda)\right) - \left(v_l(\lambda) \circ \partial\right) A_{\bar{l}}(x) \right]_{x=q_l(\lambda)} \\ &= e_l \left\langle F_{\bar{l}}\left(q_l(\lambda)\right) v_l(\lambda) \right\rangle_1 \end{aligned} \tag{17}$$

where

$$F_{\bar{l}}(x) = \partial \wedge A_{\bar{l}}(x) = \sum_{\substack{k\\k\neq l}} e_k \int \mathrm{d}\kappa v_k(\kappa) \wedge \partial\left(G_{sym}\left(x - q_k(\kappa)\right)\right) \tag{18}$$

is the Faraday of all but the l^{th} charge. Given N charges, Eq. (17) is a system of N coupled integro-differential equations, and is the dynamics predicted by the 'traditional' classical Fokker theory, regardless of the veracity or otherwise of the Wheeler-Feynman absorber mechanism. The corresponding 1st-quantized system is

$$\left(i\hbar\partial + e_l A_{\bar{l}}(x) + m_l\right)\psi_l(x) = 0 \tag{19}$$

where

$$A_{\bar{l}}(x) := \sum_{\substack{k\\k\neq l}} e_k \int \mathrm{d}^4 x G_{sym}\left(x - q_k(\kappa)\right) \left\langle \psi_k(x) \bar{\psi}_k(x) \right\rangle_1 \tag{20}$$

The 2nd quantized system can be obtained from (19) and (20) by introduction of anti-commuting operators acting on a state space.

In line with the findings and subsequent argument made above we now suppose that field-theoretical (as opposed to Fokker) electrodynamics is emergent from a Fokker theory in which net time-symmetric fields (i.e. wherein a Wheeler-Feynman-type mechanism does not entirely destroy the time-symmetry of the fields intrinsic to each charge) play a crucial role. Restricting attention to electrons and positrons, the underlying action is now presumed to be

$$I_l = -\frac{1}{2} m_e \int \mathrm{d}\lambda c_l(\lambda) v_l^2(\lambda) - \int \mathrm{d}^4 x A_{\bar{l}}(x) \circ j_l(x) = -\int \mathrm{d}\lambda \left\{ \frac{1}{2} m_e c_l(\lambda) v_l^2(\lambda) + e_l A_{\bar{l}}\left(q_l(\lambda)\right) \circ v_l(\lambda) \right\} \tag{21}$$

The associated Euler equations are

$$m_e \frac{\mathrm{d}}{\mathrm{d}\lambda} c_l(\lambda) v_l(\lambda) = e_l \left\langle F_{\bar{l}}\left(q_l(\lambda)\right) v_l(\lambda) \right\rangle_1 ; \quad v_l^2(\lambda) = 0 \tag{22}$$

Again this is a system of N coupled integro-differential equations. For convergence with standard theory we require that the dynamics predicted by (22) of any one of these charges will conform either to (17) or (19) wherein $F_{\bar{l}}(x)$ and $A_{\bar{l}}(x)$ are entirely anti-symmetric relative to the worldline of the l^{th} charge. Thus, if, in a particular instance, $F_{\bar{l}}(x)$ in (22) is entirely time-symmetric, then we must show that the dynamics predicted by (23) is equivalently captured by

$$m_e \frac{\mathrm{d}}{\mathrm{d}\lambda} \frac{v_l(\lambda)}{\sqrt{v_l^2(\lambda)}} = 0 \tag{24}$$

or by

$$(i\hbar\partial + m_l)\psi_l(x) = 0 \tag{25}$$

Clearly, the velocity in (24) cannot be that determined from (22). Thus if the classical relativistic system emerges from (22), the position of the classical particle can only approximate to the actual position as predicted by (22). If instead the quantum system is found to emerge from (22), then we might reasonably expect that the velocity eigenvalue extracted according to the rules of QM from the wavefunction satisfying (25) will be the same as that predicted by (22).

2.5 SUMMARY

To summarize the above: a viable Fokker theory with a dynamic mass that admits a role for net time-symmetric fields must be equivalent either to the classical or quantum theories wherein there is no explicit role for such fields.

2.6 RADIATION

The above is concerned with the emergence of (24) or (25) from a more fundamental Fokker-type theory in which the mass is dynamic.[8] But it does not address the origin of radiation. If in fact (24) or (25) *does* emerge as predicted, it remains to find within the Fokker theory an alternative to the vacuum degrees of freedom of traditional field theory in order to explain the exclusively retarded signature of radiation.

8. The reader is reminded that this state of affairs appears to be forced on us once the Fokker paradigm is adopted – unless there is perfect extrinsic cancellation of the advanced component of the intrinsic time-symmetric potential.

3 ORIGIN OF THE MASS OF THE CHARGE

3.1 DYNAMIC REST MASS

Eq. (A55) was interpreted as a static constraint on the electron mass. Introducing

$$\tilde{\mathbf{S}}_{ij}(\omega) = -m_e\tilde{\mathbf{M}}_{ij}(\omega) = -\left(1-\delta_{ij}\right)\frac{e_i e_j \cos\left(\omega r_{ij}\right)}{4\pi r_{ij}}\mathbb{U}\left(\mathbf{x}_{ij}\right) \tag{26}$$

(A6) can be written

$$\tilde{\mathbf{S}}(\omega)\tilde{\mathbf{a}}(\omega) = m_e\tilde{\mathbf{a}}(\omega) \tag{27}$$

The calculation in the previous section showed that m_e is an eigenvalue of $\tilde{\mathbf{S}}(\omega)$ with values given by (A47). Since $\tilde{\mathbf{S}}(\omega)$ is symmetric the m_e is real. Squaring to remove the sign ambiguity:

$$\tilde{\mathbf{S}}^2(\omega)\tilde{\mathbf{a}}(\omega) = m_e^2\tilde{\mathbf{a}}(\omega) \tag{28}$$

m_e^2 is the *N*-fold-degerente eigenvalue of $\tilde{\mathbf{S}}^2(\omega)$. The mass so computed turned out to be constant (not a function of ω, as one might suppose from the structure of (27)) because $\det\left(1+\tilde{\mathbf{M}}(\omega)\right)$ was found to be independent of ω. In the calculation of $\det\left(1+\tilde{\mathbf{M}}(\omega)\right)$ the replacement of a sum over charges with an expectation had the effect of removing all oscillatory terms, and therefore any dependence on ω.[9]

Whether or not the mass is strictly constant or time-varying there is no *physical* mechanism in the classical model investigated in Appendix A whereby the mass is determined by the distribution of charges. We are forced to the position that in a Fokker theory with net time-symmetric acceleration fields[10] the electron mass cannot be dominated either by an intrinsic *mechanical* inertia,[11] nor by a 'fixed' electromagnetic mass due to self-action, but is the consequence in large part if not entirely of the collective electromagnetic response of other charges. This will require the replacement of the traditional classical mechanical action with an action expressing a dynamic mass that is somehow responsive to the environment.

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9. If we had used instead an actual distribution of charges the determinant would have turned out to depend on ω, though probably only weakly so. We could not however simply write $m_e \to m_e(\omega)$ because a time-dependent mass would change the equations of motion, and (A1) would no longer apply.

10. By *net* time-symmetric, we mean, after allowing for superposition of the fields of multiple charges. There is a net time-symmetric acceleration field if the time-symmetric acceleration fields intrinsically to each individual charge is not entirely converted to a retarded (radiation) field by superposition of the fields of all other charges.

11. Unless one takes the view that the conformal scale factor is actually the dynamic aspect demanded by the Fokker interaction. To do so would be to equate the dynamic effects of the time-symmetric fields with gravity. Although in the end that is approximately the path one is forced to take, at this stage we concerned only to show that the mass must be dynamic, even if gravity did not exist.

3.2 LOCAL COMPONENTS

In a pre-Higgs treatment the observed mass derives from a mechanical part and an electromagnetic part. In the classical theory the latter is due exclusively to self-action, which may be written

$$I_{self} = -\frac{e^2}{|\varepsilon|}\int \mathrm{d}t\sqrt{v^2} \tag{29}$$

where, at some point in a calculation one intends to let $|\varepsilon| \to 0$. In a purely classical theory in which self-action is considered the observed mass is taken to be electromagnetic (self) mass offset by a suitably chosen mechanical mass such that the total is finite and matches the observed value. This is possible because the mechanical action has the same form as (29). In Quantum Theory there are singular contributions to the 4-momentum (not just the rest-mass) due to interaction with the vacuum EM field. These require their own renormalization, relying again on a distinction between bare and observed parameters of the theory.

All such components can be regarded as local to the charge in the sense that they do not depend on the environment of distant charges, to be constrasted with the dynamic electromagnetic mass introduced above.

A theory in which mass derives entirely from interaction with external charges one must still deal with singular self-action. One possibility is simply to deny self-action altogether. In that case the bare charge might conceivably move at any speed. However, since the *slightest* electric field will accelerate such a charge to the speed of light, in practice such a charge will *always* be moving at light speed. It is doubtful however that self-action can be excluded because it appears to be an unavoidable side-effect of admitting pair creation and destruction in quantum field theory.[12] We conclude that if self-action is generally admitted then either circumstances must be found in which the self-energy is zero or finite, or else the renormalization procedures must be adapted so that the total of the local components of mass (mechanical and electromagnetic) is zero. Since the latter is contrived[13] we discuss only implementations of the former.

12. Since it is easy to implement in the Direct Action paradigm the exclusion of self-action was seen, historically, as a reason for favoring self-action over field theory. There were two problems with this view: as subsequently pointed out by Feynman self-action appears to be an unavoidable side-effect of addmitting pair creation and destruction in quantum field theory. Further: as shown by [], it is not difficult to modify the classical Maxwell (field) theory to exclude self-action; Direct Action is not necessary for that purpose.

13. But is it any more contrived than traditional renormalization?

3.3 LIGHT-SPEED CHARGE

We observe that for a particular form of the action, (see Ibison, 2008: arxiv.org/abs/0810.4618), the electromagnetic self-mass is zero when the charge is moving at precisely light speed, and infinite otherwise.[14] A light-speed charge with zero mechanical mass (if the latter was finite then the mass would require renormalization) is therefore a candidate for a charge capable of acquiring a finite dynamic mass through interaction with distant charges. One method of enforcing light-speed motion of via the supplemental action

$$I_m = -\frac{1}{2} m_e \int \mathrm{d}t\, f(t) v^2(t) \,. \tag{30}$$

$f(t)$ is a Lagrange multiplier whose variation gives $v^2(t)=1$. We are encouraged by the fact, in compliance with the discussion above, that it appears as if the erstwhile zero-mass bare charge acquires mass dynamically - in the end through interaction with external fields. In this context, i.e. in a Fokker paradigm, we must take these to be the fields mediating time-symmetric interactions. Any 'emergent' apparently time anti-symmetric fields (i.e. radiation) can presumably be accommodated subsequently, after the 'mathematical dust has settled' and the charge deemed to acquired a definite mass from its environment.[15]

3.4 ON THE COEXISTANCE OF MECHANICAL AND ELECTROMAGNETIC MASS

In FRW Cosmological spacetime expressed in conformal coordinates the mechanical mass is augmented by a time-dependent scale factor $a(t)$. The classical action is

$$I_{mech} = -m_e \int \mathrm{d}t\, a(t) \sqrt{v^2(t)} \tag{31}$$

whilst the Dirac equation is

$$i\hbar \partial\!\!\!/ \psi(x) = a(t) m_e \psi(x) \tag{32}$$

(The 'spin connection' term results in a pure phase adjustment to the wavefunction.) As we have said, in both cases the effect of the scale factor is to make the mass appear as if dynamic.

In a conformally-expressed spacetime the EM fields are completely unaffected by the scale factor (in this case the expansion); the EM action for the fields and the field-current interaction can be written as if in Minkowski

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14. We ignore the possibility of superluminal motion, which has been discussed in this context elsewhere.

15. We cannot say "after same steady-state has been acheived", because that would imply a development in time. In the Fokker paradigm the 'back-and-forth' of an iterative calculation of the effects of the invironment involves the propagation of influences backwards and forwards in time. These are a just a means to calculation and physically invisible. Observation is always of the 'final' state of affairs of such a calculation.

spacetime.[16] Of course there is no new physics in switching from the traditional RW coordinate system to a conformal system. An advantage of the latter however is that highlights these different behaviors; the same point can be made in a traditional RW coordinate system, but it is messier because both the matter and EM actions depend (differently) on the scale factor.

A problem is that due to the presence of the conformal scale factor in the mechanical mass but not the electromagnetic mass the two parts scale differently with cosmological expansion.[17] In theories in which the electromagnetic mass is infinite, mass renormalization to a finite observed value involves subtraction of another infinite quantity. For this to work both parts must scale with expansion identically otherwise the renormalization scheme would fail. Generally this issue is not considered, perhaps because it seems minor when compared with the problem of removing an infinity. These considerations apply when the mechanical mass is replaced with the Higgs mechanism: only if the Higgs field is conformally invariant will mass renormalization survive the effects of cosmological expansion. By contrast a theory (e.g. as described above) in which the mass is electromagnetic and finite does not suffer from this problem, provided the mechanical part is expunged.

16. In the conformal system there is no Cosmological red-shift of radiation, there is instead a progressive blue shift of matter. Observationally these are indistinguishable.

17. EM action is scale invariant but the mechanical action is not.

APPENDIX A NON-RELATIVISTIC NORMAL MODES IN MINKOWSKI SPACETIME

I. NO SECULAR DRIFT

We start from

$$\mathbf{a}_i(t) = -\frac{1}{8\pi m_e}\sum_{\substack{j=1\\ j\neq i}}^{N}\frac{e_i e_j}{r_{ij}}\mathbb{U}(\mathbf{x}_{ij})\left(\mathbf{a}_j(t+r_{ij}) - \mathbf{a}_j(t-r_{ij})\right) \tag{A1}$$

where $\mathbf{a}_i(t) = \ddot{\mathbf{x}}_i(t)$ is the ordinary acceleration of a typical electron whose mean position is $\mathbf{x}_i$, and instantaneous position is $\mathbf{x}_i + \mathbf{x}_i(t)$ and $\mathbf{x}_{ij} = \mathbf{x}_i - \mathbf{x}_j$. Eq. (A1) is a linear difference equation in the $\mathbf{a}_i(t)$. Let us collect the individual vectors together into $N\otimes 3$ vector of vectors, where N is the number of charges:

$$\mathbf{a}(t) = \left(\mathbf{a}_1(t), \mathbf{a}_2(t), \ldots, \mathbf{a}_N(t)\right) \tag{A2}$$

Eq. (A1) can then be written in matrix form as

$$\left(1 + \hat{\mathbf{M}}(t)\right)\mathbf{a}(t) = \mathbf{0} \tag{A3}$$

where $\hat{\mathbf{M}}(t)$ is an $N\otimes N$ matrix of $3\otimes 3$ matrixes:

$$\left(\hat{\mathbf{M}}(t)\right)_{ij} = \left(1-\delta_{ij}\right)\frac{e_i e_j}{8\pi m_e r_{ij}}\mathbb{U}(\mathbf{x}_{ij})\left(\hat{\mathrm{E}}(r_{ij}) + \hat{\mathrm{E}}(-r_{ij})\right) \tag{A4}$$

and $\hat{\mathrm{E}}$ is a shift operator acting on the time:

$$\hat{\mathrm{E}}(r)\, f(t) = f(t+r) \tag{A5}$$

In the Fourier domain

$$\left(1 + \tilde{\mathbf{M}}(\omega)\right)\tilde{\mathbf{a}}(\omega) = \mathbf{0} \tag{A6}$$

where

$$\begin{aligned}\tilde{\mathbf{a}}(\omega) &= \left(\tilde{\mathbf{a}}_1(\omega), \tilde{\mathbf{a}}_2(\omega), \ldots, \tilde{\mathbf{a}}_N(\omega)\right)\\ \tilde{\mathbf{M}}_{ij}(\omega) &= \left(1-\delta_{ij}\right)\frac{e_i e_j \cos(\omega r_{ij})}{4\pi m_e r_{ij}}\mathbb{U}(\mathbf{x}_{ij})\end{aligned} \tag{A7}$$

The solutions $\tilde{\mathbf{a}}(\omega)$ of (A6) are the self-consistent accelerations and can be expressed as a sum over the set of vectors that form the null space of $1+\tilde{\mathbf{M}}(\omega)$. Normal modes solutions exist only if

$$\det\left(1 + \tilde{\mathbf{M}}(\omega)\right) = 0 \tag{A8}$$

where $\tilde{\mathbf{M}}(\omega)$ is very large, off-diagonal, and Hermitian. In the following we will regard $\tilde{\mathbf{a}}(\omega)$ as a straight-forward vector of $3N$ components, and $\tilde{\mathbf{M}}(\omega)$ as an $3N\otimes 3N$ matrix. Using the traditional re-formulation and suppressing the frequency argument the determinant is

$$\det\left(1+\tilde{\mathbf{M}}\right) = \exp\left(\mathrm{tr}\left(\log\left(\left(1+\tilde{\mathbf{M}}\right)\right)\right)\right) = \exp\left(\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}\mathrm{tr}\left\{\tilde{\mathbf{M}}^n\right\}\right) \tag{A9}$$

where

$$\mathrm{tr}\left\{\tilde{\mathbf{M}}^m\right\} \approx \left(\frac{e^2}{4\pi m_e}\right)^m \sum_{\substack{n_1=1\\n_1\neq n_2}}^{N}\sum_{\substack{n_2=1\\n_2\neq n_3}}^{N}\cdots\sum_{\substack{n_m=1\\n_m\neq n_1}}^{N}\left\{\begin{array}{l}\dfrac{\cos\left(\omega r_{n_1,n_2}\right)}{r_{n_1,n_2}}\dfrac{\cos\left(\omega r_{n_2,n_3}\right)}{r_{n_2,n_3}}\cdots\dfrac{\cos\left(\omega r_{n_m,n_1}\right)}{r_{n_m,n_1}}\\ \times\,\mathrm{tr}\left\{\mathbb{U}\left(\mathbf{x}_{n_1,n_2}\right)\mathbb{U}\left(\mathbf{x}_{n_2,n_3}\right)\ldots\mathbb{U}\left(\mathbf{x}_{n_m,n_1}\right)\right\}\end{array}\right\} \tag{A10}$$

Clearly $\mathrm{tr}\left(\tilde{\mathbf{M}}\right)=0$ because the diagonal elements are zero. At the next order,

$$\mathrm{tr}\left\{\tilde{\mathbf{M}}^2\right\} = \left(\frac{e^2}{4\pi m_e}\right)^2 \sum_{\substack{i,j\\i\neq j}}\left(\frac{\cos\left(\omega r_{ij}\right)}{r_{ji}}\right)^2 \mathrm{tr}\left\{\mathbb{U}\left(\mathbf{x}_{ij}\right)\right\} \tag{A11}$$

Using that

$$\mathrm{tr}\left\{\mathbb{U}\left(\mathbf{x}_{n_1,n_2}\right)\right\} \approx 3\left\langle\mathbb{U}\left(\mathbf{x}_{n_1,n_2}\right)\right\rangle = 2 \tag{A12}$$

and

$$\sum_{\substack{i,j\\i\neq j}}\left(\frac{\cos\left(\omega r_{ij}\right)}{r_{ji}}\right)^2 \approx N^2\left\langle\left(\frac{\cos\left(\omega r\right)}{r}\right)^2\right\rangle \approx \frac{1}{2}N^2\left\langle\frac{1}{r^2}\right\rangle = \frac{3N^2}{2R^2} \tag{A13}$$

Then

$$\mathrm{tr}\left\{\tilde{\mathbf{M}}^2\right\} = 3Nx^2 \tag{A14}$$

where

$$x := \frac{\sqrt{N}e^2}{4\pi m_e R} \tag{A15}$$

$$\mathrm{tr}\left\{\tilde{\mathbf{M}}^4\right\} \approx \left(\frac{e^2}{4\pi m_e}\right)^4 \sum_{\substack{i,j,k,l=1\\i\neq j,j\neq k,k\neq l,l\neq i}}^{N}\left\{\begin{array}{l}\dfrac{\cos\left(\omega r_{ij}\right)}{r_{ij}}\dfrac{\cos\left(\omega r_{jk}\right)}{r_{jk}}\dfrac{\cos\left(\omega r_{kl}\right)}{r_{kl}}\dfrac{\cos\left(\omega r_{li}\right)}{r_{li}}\\ \times\,\mathrm{tr}\left\{\mathbb{U}\left(\mathbf{x}_{ij}\right)\mathbb{U}\left(\mathbf{x}_{jk}\right)\mathbb{U}\left(\mathbf{x}_{kl}\right)\mathbb{U}\left(\mathbf{x}_{li}\right)\right\}\end{array}\right\} \tag{A16}$$

In order to obtain estimates for the trace of an arbitrary power it will be useful to split $\mathrm{tr}\left\{\tilde{\mathbf{M}}^m\right\}$ into synchronous and asynchronous parts:

$$\mathrm{tr}\left\{\tilde{\mathbf{M}}^m\right\} = \mathrm{sync}\left[\mathrm{tr}\left\{\tilde{\mathbf{M}}^m\right\}\right] + \mathrm{async}\left[\mathrm{tr}\left\{\tilde{\mathbf{M}}^m\right\}\right] \tag{A17}$$

The synchronous part is comprised of terms wherein each fourier factor $\cos\left(\omega r_{ij}\right)$ appears as a square. Observing that there are $\left(N-1\right)^m \approx N^m$ additive terms in $\mathrm{tr}\left\{\tilde{\mathbf{M}}^m\right\}$, there will be approximately $N^{1+m/2}$ squared terms when m is even. At fourth order

$$\mathrm{sync}\left[\mathrm{tr}\left\{\tilde{\mathbf{M}}^4\right\}\right] \approx \left(\frac{e^2}{4\pi m_e}\right)^4 N^3\left\langle\left(\frac{\cos\left(\omega r\right)}{r}\right)^2\right\rangle^2 T_4 \tag{A18}$$

T is the trace of the product of the synchronous 3x3 matrixes. There are distinguishably different orderings of these matrixes corresponding to different possible topologies in the scattering diagram. (Unlike the scalar terms $\cos(\omega r_{ij})/r_{ij}$, the matrix products are distinguishable because the matrixes do not commute.) Corresponding to two topologies at 4th order we have

$$T_4 = \frac{1}{w_1 + w_2} \mathrm{tr}\left\{ w_1 \left\langle \mathbb{U}(\mathbf{x}_{12})\mathbb{U}(\mathbf{x}_{23})\mathbb{U}(\mathbf{x}_{32})\mathbb{U}(\mathbf{x}_{21}) \right\rangle + w_2 \left\langle \mathbb{U}(\mathbf{x}_{12})\mathbb{U}(\mathbf{x}_{21})\mathbb{U}(\mathbf{x}_{13})\mathbb{U}(\mathbf{x}_{31}) \right\rangle \right\} \tag{A19}$$

where w_1, w_2 are weights determined by the frequency of occurance of each of the two possibilities in the sum in (A16). Since however

$$\mathbb{U}^2(\mathbf{x}_{ij}) = \mathbb{U}^T(\mathbf{x}_{ij}) = \mathbb{U}(\mathbf{x}_{ij}) = \mathbb{U}(\mathbf{x}_{ji}) \tag{A20}$$

and $\mathrm{tr}\{\mathbb{A}\mathbb{B}\} = \mathrm{tr}\{\mathbb{B}^T \mathbb{A}^T\}$ the trace of each term is the same, and

$$T_4 = \mathrm{tr}\left\{ \left\langle \mathbb{U}(\mathbf{x}_{12})\mathbb{U}(\mathbf{x}_{23}) \right\rangle \right\} = 3\left(\frac{2}{3}\right)^2 \tag{A21}$$

Consequently (A19) reads

$$\mathrm{sync}\left[\mathrm{tr}\left\{ \tilde{\mathbf{M}}^4 \right\} \right] \approx \left(\frac{e^2}{4\pi m_e} \right)^4 N^3 \left(\frac{1}{2}\right)^2 \left(\frac{3}{R^2}\right)^2 3\left(\frac{2}{3}\right)^2 = 3Nx^4 \tag{A22}$$

At 6th order

$$\mathrm{sync}\left[\mathrm{tr}\left\{ \tilde{\mathbf{M}}^6 \right\} \right] \approx N^4 \left(\frac{e^2}{4\pi m_e} \right)^6 \left(\frac{1}{2}\right)^3 \left(\frac{3}{R^2}\right)^3 T_6 \tag{A23}$$

where

$$T_6 = \frac{1}{w_1 + w_2 + w_3} \mathrm{tr}\left\{ \begin{array}{l} w_1 \left\langle \mathbb{U}(\mathbf{x}_{12})\mathbb{U}(\mathbf{x}_{23})\mathbb{U}(\mathbf{x}_{34})\mathbb{U}(\mathbf{x}_{43})\mathbb{U}(\mathbf{x}_{32})\mathbb{U}(\mathbf{x}_{21}) \right\rangle \\ +w_2 \left\langle \mathbb{U}(\mathbf{x}_{12})\mathbb{U}(\mathbf{x}_{21})\mathbb{U}(\mathbf{x}_{13})\mathbb{U}(\mathbf{x}_{34})\mathbb{U}(\mathbf{x}_{43})\mathbb{U}(\mathbf{x}_{31}) \right\rangle \\ +w_3 \left\langle \mathbb{U}(\mathbf{x}_{12})\mathbb{U}(\mathbf{x}_{21})\mathbb{U}(\mathbf{x}_{13})\mathbb{U}(\mathbf{x}_{31})\mathbb{U}(\mathbf{x}_{14})\mathbb{U}(\mathbf{x}_{41}) \right\rangle \end{array} \right\} \tag{A24}$$

Using (A21) leads to

$$T_6 = \frac{1}{w_1 + w_2 + w_3} \mathrm{tr}\left\{ \begin{array}{l} w_1 \left\langle \mathbb{U}(\mathbf{x}_{12})\mathbb{U}(\mathbf{x}_{23})\mathbb{U}(\mathbf{x}_{34})\mathbb{U}(\mathbf{x}_{32}) \right\rangle \\ +w_2 \left\langle \mathbb{U}(\mathbf{x}_{12})\mathbb{U}(\mathbf{x}_{13})\mathbb{U}(\mathbf{x}_{34})\mathbb{U}(\mathbf{x}_{31}) \right\rangle \\ +w_3 \left\langle \mathbb{U}(\mathbf{x}_{12})\mathbb{U}(\mathbf{x}_{13})\mathbb{U}(\mathbf{x}_{14}) \right\rangle \end{array} \right\} \tag{A25}$$

Performing the orientation average over the coordinates pairs that occur just once in each additive term leads to

$$T_6 = \frac{1}{w_1 + w_2 + w_3} \mathrm{tr}\left\{ \begin{array}{l} w_1 \left(\frac{2}{3}\right)^2 \left\langle \mathbb{U}(\mathbf{x}_{23})\mathbb{U}(\mathbf{x}_{32}) \right\rangle \\ +w_2 \left(\frac{2}{3}\right)^2 \left\langle \mathbb{U}(\mathbf{x}_{13})\mathbb{U}(\mathbf{x}_{31}) \right\rangle \\ +w_3 \left(\frac{2}{3}\right)^3 \end{array} \right\} = \frac{1}{w_1 + w_2 + w_3} \mathrm{tr}\left\{ \begin{array}{l} w_1 \left(\frac{2}{3}\right)^2 \left\langle \mathbb{U}(\mathbf{x}_{23}) \right\rangle \\ +w_2 \left(\frac{2}{3}\right)^2 \left\langle \mathbb{U}(\mathbf{x}_{13}) \right\rangle \\ +w_3 \left(\frac{2}{3}\right)^3 \end{array} \right\} \tag{A26}$$

Performing the remaining orientation average gives

$$T_6 = 3\left(\frac{2}{3}\right)^3 \tag{A27}$$

Used in (A23) this gives

$$\text{sync}\left[\text{tr}\left\{\tilde{\mathbf{M}}^6\right\}\right] \approx 3Nx^6 \tag{A28}$$

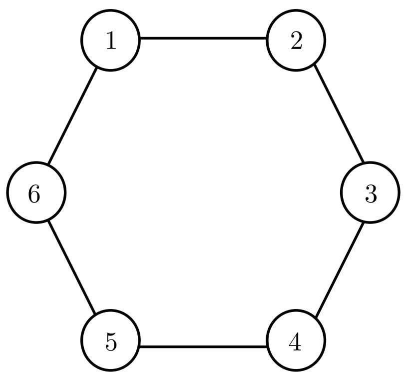
We notice that regardless of the topology (and consequent ordering of the matrixes)

$$T_{2m} = 3\left(\frac{2}{3}\right)^m \tag{A29}$$

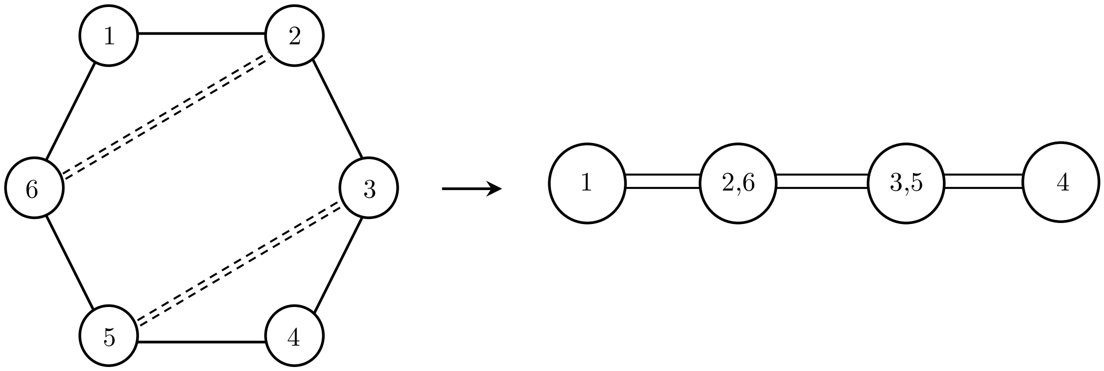
Consequently

$$\text{sync}\left[\text{tr}\left\{\tilde{\mathbf{M}}^{2m}\right\}\right] \approx 3Nx^{2m} \tag{A30}$$

The relative strength of the asynchronous terms can be inferred from a graphical depiction of the scattering process underlying (A30). Consider at first the term $m=6$. Before separating $\text{tr}\left\{\tilde{\mathbf{M}}^6\right\}$ into synchronous and asynchronous terms the scattering is a sum over all possible positions of the nodes numbered 1 to 6 in the hexagon

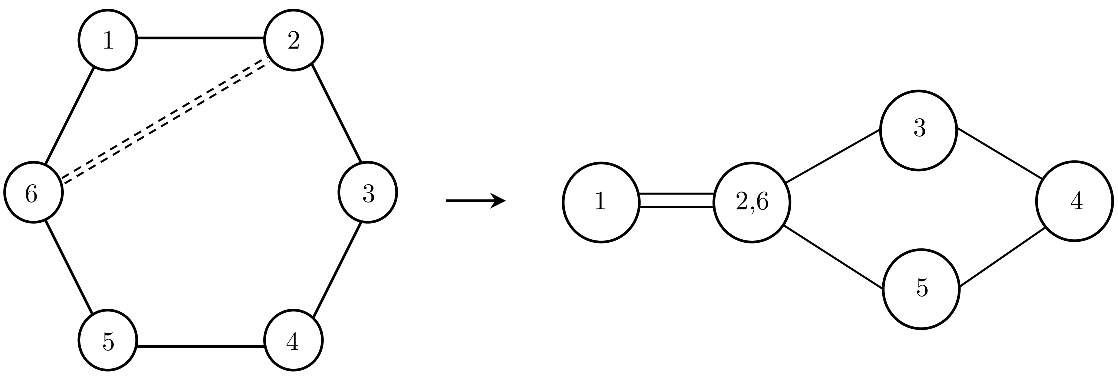


Clearly there are approximately N^6 terms. (We will be comparing different powers of *N*; due to the size of *N* we can ignore adjustments prohibiting self-action, and also complications in counting the terms arising from the traces of the 3x3 matrixes.) Synchrony demands that all links occur in pairs. At $m=6$ this is achieved by identifying *two* pairs of nodes. For example identifying nodes 2 and 6, and nodes 3 and 5, leads to



Clearly there are N^4 terms having the topology of the new graph. Writing this as N^{6-2}, 6 is the number of nodes in the original polygon, and 2 is the number of identifications. This result is confirmed by (A28) wherein $\text{sync}\left[\text{tr}\left\{\tilde{\mathbf{M}}^6\right\}\right] \propto Nx^6 \propto N^4$. Note that adjacent nodes cannot be merged, since this would be equivalent to permitting self-interaction.

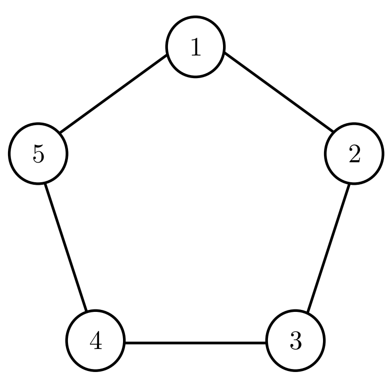
The most asynchronous topology at $m=6$ leaves all six nodes unrelated. The expectation of the trace is zero and the standard deviation goes as $\left(\sqrt{N}\right)^6 = N^3$. Given the size of N this can be ignored compared to the synchronous contribution. But there are intermediate topologies that involve both synchronous and asynchronous loops. At $m=6$ for example we must also consider



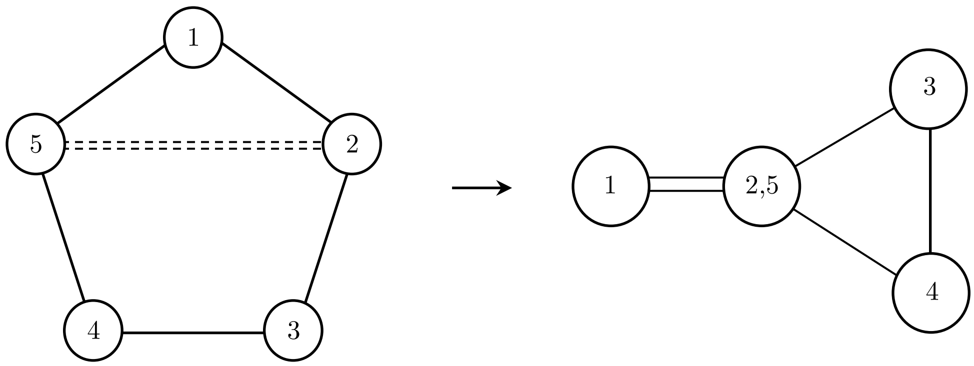
Here there are N^2 synchronous terms multiplied by N^3 asynchronous terms. The expected value of the latter is zero whilst the expected value of their variance is proportional to N^3. Consequently they contribute a factor proportional to $z\sqrt{N^3}$, where z is a zero mean Gaussian random variable (independent of N). The contribution from this graph therefore goes as $N^2 \times z\sqrt{N^3} = zN^{7/2}$. Since this is the dominant asynchronous contribution at $m=6$ we have

$$\frac{\text{async}\left[\text{tr}\left\{\tilde{\mathbf{M}}^6\right\}\right]}{\text{sync}\left[\text{tr}\left\{\tilde{\mathbf{M}}^6\right\}\right]} \propto \frac{z}{\sqrt{N}} \tag{A31}$$

We use $m=5$ as an example of odd m. There are no perfectly synchronous contributions when m is odd. The graph depicting the maximally asynchronous contribution is



There are N^5 instances of this graph, which contributes therefore $zN^{5/2}$. Consider also the 'partially synchronous' graph



Here there are N^2 synchronous terms multiplied by N^2 asynchronous terms. The expected value of the latter is zero whilst the expected value of their variance is proportional to N^2. Consequently they contribute a factor proportional to zN. The contribution from instances of this graph therefore goes as $N^2 \times zN = zN^3$ - a factor of $\sqrt{N}$ stronger than the maximally asynchronous contribution.

The general rule for arbitrary m is easily inferred from the above examples. A polygon with m vertexes has m links. When m is even the (fully) synchronous graph has the same number of links as the polygon that represents the maximally asynchronous contribution. In the synchronous graph these occur in pairs, which contains therefore $m/2$ double links. The nodes and double links are arranged in a simple chain, which contains therefore $m/2+1$ nodes. Therefore

$$\text{sync}\left[\text{tr}\left\{\tilde{\mathbf{M}}^{2m}\right\}\right] \propto N^{m+1} \tag{A32}$$

Staying with even m, there are no 'partially synchronous' graphs of the kind depicted in Fig. [] at $m=2$ and $m=4$. ($m=2$ is synchronous, $m=4$ is either synchronous or maximally asynchronous).

$$\text{async}\left[\text{tr}\left\{\tilde{\mathbf{M}}^{2}\right\}\right] = 0, \quad \text{async}\left[\text{tr}\left\{\tilde{\mathbf{M}}^{4}\right\}\right] \propto zN^2 \tag{A33}$$

For $m \geq 6$ and even, the 'partially synchronous' graph making the largest contribution is a double-linked chain attached to a (single-linked) square. The 3 nodes in the square that are not part of the chain contribute a factor $zN^{3/2}$. There are $m-4$ links in the double-linked chain, which has, therefore $(m-4)/2+1 = m/2-1$ nodes. Therefore the chain contributes a factor $N^{m/2-1}$. It follows that

$$\text{async}\left[\text{tr}\left\{\tilde{\mathbf{M}}^{2m}\right\}\right] \propto zN^{3/2} \times N^{m-1} = zN^{m+1/2} \tag{A34}$$

and therefore

$$\frac{\text{async}\left[\text{tr}\left\{\tilde{\mathbf{M}}^{2m}\right\}\right]}{\text{sync}\left[\text{tr}\left\{\tilde{\mathbf{M}}^{2m}\right\}\right]} \propto \frac{z}{\sqrt{N}} \tag{A35}$$

independent of m provided $m \geq 3$.

When m is odd and $m \geq 5$ the 'partially synchronous' graph making the largest contribution is a double-linked chain attached to a (single-linked) triangle. The 2 nodes in the square that are not part of the chain contribute a factor zN. There are $m-3$ links in the double-linked chain, which has, therefore $(m-3)/2+1=(m-1)/2$ nodes. Therefore the chain contributes a factor $N^{(m-1)/2}$. It follows that

$$\text{async}\left[\text{tr}\left\{\tilde{\mathbf{M}}^{2m+1}\right\}\right] \propto zN \times N^m = zN^{m+1} \tag{A36}$$

Combining this with (A34) gives

$$\text{async}\left[\text{tr}\left\{\tilde{\mathbf{M}}^{m}\right\}\right] \propto zN^{(m+1)/2} \tag{A37}$$

provided $m \geq 5$. At $m=1$ $\text{tr}\{\tilde{\mathbf{M}}\}=0$ (due to the absence of self-interaction). At $m=3$ there is no way to make a chain (adjacent nodes cannot be merged), and therefore

$$\text{async}\left[\text{tr}\left\{\tilde{\mathbf{M}}^{3}\right\}\right] \propto zN^{3/2} \tag{A38}$$

In summary

m	$\text{async}\left[\text{tr}\left\{\tilde{\mathbf{M}}^{m}\right\}\right]$	$\text{sync}\left[\text{tr}\left\{\tilde{\mathbf{M}}^{m}\right\}\right]$
1	0	0
2	0	$3Nx^2 \propto N^2$
3	$zN^{3/2}$	0
4	zN^2	$3Nx^4 \propto N^3$
≥ 5	$zN^{(m+1)/2}$	$\begin{cases} 3Nx^m \propto N^{m/2+1} & m \text{ even} \\ 0 & m \text{ odd} \end{cases}$

Note that the synchronous contribution dominate at all even powers.

Using (A30) in (A9) gives

$$\det\left(1+\tilde{\mathbf{M}}(\omega)\right) = f(x)\exp\left(-\frac{3N}{2}\sum_{m=1}^{\infty}\frac{x^{2m}}{m}\right) \tag{A39}$$

where $f(x)$ contains the asynchronous contributions. Summing the series one has

$$\det\left(1+\tilde{\mathbf{M}}(\omega)\right) = f(x)\exp\left(\frac{3N}{2}\ln\left(1-x^2\right)\right) = f(x)\left(1-x^2\right)^{3N/2} \tag{A40}$$

Unless $f(x)$ contains a factor that dominates the behavior of the whole product near $x^2=1$ (e.g. $f(x)=x/\left(1-x^2\right)^{3N/2}$), the determinant vanishes when $x^2=1$. (This is unique only if $f(x)=0$ has no solutions for real positive x orher than at $x^2=1$.)

In order to investigate further, consistent with Table [] let

$$\begin{aligned}
&\operatorname{async}\left[\operatorname{tr}\left\{\tilde{\mathbf{M}}^1\right\}\right]=\operatorname{async}\left[\operatorname{tr}\left\{\tilde{\mathbf{M}}^2\right\}\right]=0\\
&\operatorname{async}\left[\operatorname{tr}\left\{\tilde{\mathbf{M}}^3\right\}\right]=\lambda z_3 x^3\\
&\operatorname{async}\left[\operatorname{tr}\left\{\tilde{\mathbf{M}}^4\right\}\right]=\lambda z_4 x^4\\
&\operatorname{async}\left[\operatorname{tr}\left\{\tilde{\mathbf{M}}^m\right\}\right]=\lambda\sqrt{N} z_m x^m;\quad m\geq 5
\end{aligned} \tag{A41}$$

where λ is a dimensionless number of order unity, and the z_m are independent sanples of a Gaussian random variable having zero mean and unit variance. Then

$$f(x)=\exp\left(-\frac{\lambda}{3}z_3x^3-\frac{\lambda}{4}z_4x^4-\lambda\sqrt{N}\sum_{m=5}^{\infty}\frac{z_mx^m}{m}\right) \tag{A42}$$

Since

$$\det\left(1+\tilde{\mathbf{M}}(\omega)\right)=0 \Rightarrow \left\langle\det\left(1+\tilde{\mathbf{M}}(\omega)\right)\right\rangle=0 \tag{A43}$$

we will be content to work with the expectation of $f(x)$:

$$\begin{aligned}
\langle f(x)\rangle &= \left\langle \exp\left(-\frac{\lambda}{3}z_3x^3-\frac{\lambda}{4}z_4x^4-\lambda\sqrt{N}\sum_{m=5}^{\infty}\frac{z_mx^m}{m}\right)\right\rangle\\
&=\frac{1}{2\pi}\int \mathrm{d}\,z_3\exp\left(-\frac{z_3^2}{2}-\frac{\lambda}{3}x^3z_3\right)\int \mathrm{d}\,z_4\exp\left(-\frac{z_4^2}{2}-\frac{\lambda}{4}x^4z_4\right)\prod_{m=5}^{\infty}\left\{\int \mathrm{d}\,z_m\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{z_m^2}{2}-\frac{\lambda\sqrt{N}x^mz_m}{m}\right)\right\}
\end{aligned} \tag{A44}$$

Using that

$$\frac{1}{\sqrt{2\pi}}\int \mathrm{d}\,z\exp\left(-\frac{z^2}{2}-\alpha z\right)=e^{\alpha^2/2} \tag{A45}$$

(A44) is

$$\begin{aligned}
\langle f(x)\rangle &= \exp\left(\frac{\lambda^2x^6}{18}+\frac{\lambda^2x^8}{32}+\frac{\lambda^2N}{2}\sum_{m=5}^{\infty}\frac{x^{2m}}{m^2}\right)\\
&=\exp\left(\frac{\lambda^2x^6}{18}+\frac{\lambda^2x^8}{32}+\frac{\lambda^2N}{2}\left(\operatorname{dilog}\left(1-x^2\right)-\sum_{m=1}^{4}\frac{x^{2m}}{m^2}\right)\right)\\
&\approx\exp\left(\frac{\lambda^2N}{2}\left(\operatorname{dilog}\left(1-x^2\right)-x^2-\frac{x^4}{4}-\frac{x^6}{9}-\frac{x^8}{16}\right)\right)\\
&=\exp\left(\frac{\lambda^2N}{2}\sum_{m=5}^{\infty}\frac{x^{2m}}{m^2}\right)
\end{aligned} \tag{A46}$$

The exponent is real and positive for all real x and therefore $\langle f(x)\rangle$ has no real roots. Consequently $x^2=1$ is the only solution of $\langle \det(1+\tilde{\mathbf{M}}(\omega))\rangle = 0$, which we will write as

$$m_e = \pm\frac{\sqrt{N}e^2}{4\pi R} \tag{A47}$$

In the electromagnetic domain (i.e. not including GR) and excluding pair creation and destruction (involving conversions between inertial mass and EM radiation) the sign of the mass is fungible with the sign of the charge, and one is at liberty to fix the sign of either. I.E. there is no loss of generality choosing

$$m_e = \frac{\sqrt{N}e^2}{4\pi R} \tag{A48}$$

II. ACCOMMODATION OF SECULAR DRIFT

The charges are presumed to move sufficiently slowly that they can be treated as stationary for the duration of the interaction under consideration here. Yet even a very small motion will cause the advanced and retarded positions of a typical distant charge to be significantly different (the average separation is of the order of the Hubble radius). To accommodate such 'secular drift', relative to each particle at a particular time the worldline of every other particle is split into two, treating the past and future segments as if belonging to two different particles. Thus it then appears there are $2N$ particles, each with its own mean position. The response of a typical charge to a time-symmetric field can then be written

$$\mathbf{a}_i(t) = -\frac{1}{8\pi m_e}\sum_{\substack{j=1\\ j\neq i}}^{N} e_i e_j \left(\mathbb{U}\left(\mathbf{x}_{ij}^{+}\right)\frac{\mathbf{a}_j\left(t+r_{ij}^{+}\right)}{r_{ij}^{+}} + \mathbb{U}\left(\mathbf{x}_{ij}^{-}\right)\frac{\mathbf{a}_j\left(t-r_{ij}^{-}\right)}{r_{ij}^{-}} \right) \tag{A49}$$

where $\mathbf{x}_{ij}^{+} = \mathbf{x}_{ji}^{-}$, though in general $\mathbf{x}_{ij}^{+} \neq \mathbf{x}_{ij}^{-}$. In the Fourier domain

$$\tilde{\mathbf{a}}_i(\omega) = -\frac{1}{8\pi m_e}\sum_{\substack{j=1\\ j\neq i}}^{N} e_i e_j \left(\mathbb{U}\left(\mathbf{x}_{ij}^{+}\right)\frac{e^{i\omega r_{ij}^{+}}}{r_{ij}^{+}} + \mathbb{U}\left(\mathbf{x}_{ij}^{-}\right)\frac{e^{-i\omega r_{ij}^{-}}}{r_{ij}^{-}} \right)\tilde{\mathbf{a}}_j(\omega) \tag{A50}$$

where

$$e^{i\omega r_{ij}^{+}} = e^{-i\omega r_{ji}^{+}}, \quad e^{i\omega r_{ij}^{-}} = e^{-i\omega r_{ji}^{-}} \tag{A51}$$

Let

$$\tilde{\mathfrak{a}}(\omega) = \left(\tilde{\mathbf{a}}_1(\omega), \tilde{\mathbf{a}}_2(\omega), \ldots, \tilde{\mathbf{a}}_N(\omega)\right) \tag{A52}$$

and

$$\tilde{\mathbf{M}}_{ij}(\omega) = \left(1-\delta_{ij}\right)\frac{e_i e_j}{8\pi m_e}\left(\mathbb{U}\left(\mathbf{x}_{ij}^{+}\right)\frac{e^{i\omega r_{ij}^{+}}}{r_{ij}^{+}} + \mathbb{U}\left(\mathbf{x}_{ij}^{-}\right)\frac{e^{-i\omega r_{ij}^{-}}}{r_{ij}^{-}} \right) \tag{A53}$$

Then

$$\left(1+\tilde{\mathbf{M}}(\omega)\right)\tilde{\mathbb{a}}(\omega)=\mathbb{0} \tag{A54}$$

and solutions exist only if

$$\det\left(1+\tilde{\mathbf{M}}(\omega)\right)=0 \tag{A55}$$

where $\tilde{\mathbf{M}}(\omega)$ is very large and off-diagonal. We use (A9), for which we will need to know

$$\operatorname{tr}\left\{\tilde{\mathbf{M}}^m(\omega)\right\}=\left(\frac{e^2}{8\pi m_e}\right)^m\sum_{\substack{n_1=1\\n_1\neq n_2}}^{N}\sum_{\substack{n_2=1\\n_2\neq n_3}}^{N}\dots\sum_{\substack{n_m=1\\n_m\neq n_1}}^{N}\operatorname{tr}\left\{\begin{array}{l}\left(\mathbb{U}\left(\mathbf{x}^+_{n_1,n_2}\right)\frac{e^{i\omega r^+_{n_1,n_2}}}{r^+_{n_1,n_2}}+\mathbb{U}\left(\mathbf{x}^-_{n_1,n_2}\right)\frac{e^{-i\omega r^-_{n_1,n_2}}}{r^-_{n_1,n_2}}\right)\\ \times\left(\mathbb{U}\left(\mathbf{x}^+_{n_2,n_3}\right)\frac{e^{i\omega r^+_{n_2,n_3}}}{r^+_{n_2,n_3}}+\mathbb{U}\left(\mathbf{x}^-_{n_2,n_3}\right)\frac{e^{-i\omega r^-_{n_2,n_3}}}{r^-_{n_2,n_3}}\right)\\ \dots\times\left(\mathbb{U}\left(\mathbf{x}^+_{n_m,n_1}\right)\frac{e^{i\omega r^+_{n_m,n_1}}}{r^+_{n_m,n_1}}+\mathbb{U}\left(\mathbf{x}^-_{n_m,n_1}\right)\frac{e^{-i\omega r^-_{n_m,n_1}}}{r^-_{n_m,n_1}}\right)\end{array}\right\} \tag{A56}$$

notice that the leading factor differs from (A10). The synchronous component is now comprised of pairs of positive and negative frequency terms formed from the cross terms multiplying out the products in (A56). A typical such term involves the factor

$$\sum_{\substack{n_2,n_3=1\\n_1\neq n_2,n_2\neq n_3}}^{N}\left\{\begin{array}{l}\mathbb{U}\left(\mathbf{x}^+_{n_1,n_2}\right)\frac{e^{i\omega r^+_{n_1,n_2}}}{r^+_{n_1,n_2}}\mathbb{U}\left(\mathbf{x}^-_{n_2,n_3}\right)\frac{e^{-i\omega r^-_{n_2,n_3}}}{r^-_{n_2,n_3}}\\+\mathbb{U}\left(\mathbf{x}^-_{n_1,n_2}\right)\frac{e^{i\omega r^-_{n_1,n_2}}}{r^-_{n_1,n_2}}\mathbb{U}\left(\mathbf{x}^+_{n_2,n_3}\right)\frac{e^{-i\omega r^+_{n_2,n_3}}}{r^+_{n_2,n_3}}\end{array}\right\}\delta_{n_3,n_1}=\sum_{\substack{n_2\\n_1\neq n_2}}^{N}\left\{\begin{array}{l}\mathbb{U}\left(\mathbf{x}^+_{n_1,n_2}\right)\frac{e^{i\omega r^+_{n_1,n_2}}}{r^+_{n_1,n_2}}\mathbb{U}\left(\mathbf{x}^-_{n_2,n_1}\right)\frac{e^{-i\omega r^-_{n_2,n_1}}}{r^-_{n_2,n_1}}\\+\mathbb{U}\left(\mathbf{x}^-_{n_1,n_2}\right)\frac{e^{i\omega r^-_{n_1,n_2}}}{r^-_{n_1,n_2}}\mathbb{U}\left(\mathbf{x}^+_{n_2,n_1}\right)\frac{e^{-i\omega r^+_{n_2,n_1}}}{r^+_{n_2,n_1}}\end{array}\right\} \tag{A57}$$

Using $\mathbf{x}^+_{ij}=\mathbf{x}^-_{ji}$ and that $\mathbb{U}^2(\mathbf{x})=\mathbb{U}(\mathbf{x})$:

$$\begin{aligned}\sum_{\substack{n_2,n_3=1\\n_1\neq n_2,n_2\neq n_3}}^{N}\left\{\begin{array}{l}\mathbb{U}\left(\mathbf{x}^+_{n_1,n_2}\right)\frac{e^{i\omega r^+_{n_1,n_2}}}{r^+_{n_1,n_2}}\mathbb{U}\left(\mathbf{x}^-_{n_2,n_3}\right)\frac{e^{-i\omega r^-_{n_2,n_3}}}{r^-_{n_2,n_3}}\\+\mathbb{U}\left(\mathbf{x}^-_{n_1,n_2}\right)\frac{e^{i\omega r^-_{n_1,n_2}}}{r^-_{n_1,n_2}}\mathbb{U}\left(\mathbf{x}^+_{n_2,n_3}\right)\frac{e^{-i\omega r^+_{n_2,n_3}}}{r^+_{n_2,n_3}}\end{array}\right\}\delta_{n_3,n_1}&=\sum_{\substack{n_2\\n_1\neq n_2}}^{N}\left\{\mathbb{U}\left(\mathbf{x}^+_{n_1,n_2}\right)\frac{1}{\left(r^+_{n_1,n_2}\right)^2}+\mathbb{U}\left(\mathbf{x}^-_{n_1,n_2}\right)\frac{1}{\left(r^-_{n_1,n_2}\right)^2}\right\}\\&\approx 2N\left\langle\mathbb{U}\left(\mathbf{x}^+_{n_1,n_2}\right)\frac{1}{\left(r^+_{n_1,n_2}\right)^2}\right\rangle\\&=\frac{4N}{R^2}\end{aligned} \tag{A58}$$

These pairings occur at the same frequency as a function of m as did the $\mathbb{U}(\mathbf{x})\left(\cos(\omega r)/r\right)^2$ terms in (A10) and therefore

$$\operatorname{sync}\left[\operatorname{tr}\left\{\tilde{\mathbf{M}}^{2m}\right\}\right]\approx 3N\left(\frac{e^2}{8\pi m_e}\right)^{2m}\left(\frac{4N}{R^2}\right)^m=3N\left(\frac{\sqrt{N}e^2}{4\pi m_e R}\right)^{2m}=3Nx^{2m} \tag{A59}$$

equalling (A30). Thus the difference in the leading factors in (A56) and (A10) is cancelled by an extra factor of 4^m due to the fact that (A58) is 4 times larger than the corresponding synchronous component taken from (A10):

$$
\begin{aligned}
\sum_{\substack{n_2,n_3=1\\ n_1\neq n_2,n_2\neq n_3}}^{N} \mathbb{U}\left(\mathbf{x}_{n_1,n_2}\right)\frac{\cos\left(\omega r_{n_1,n_2}\right)}{r_{n_1,n_2}}\mathbb{U}\left(\mathbf{x}_{n_2,n_3}\right)\frac{\cos\left(\omega r_{n_2,n_3}\right)}{r_{n_2,n_3}}\delta_{n_3,n_1} &= \sum_{\substack{n_2\\ n_1\neq n_2}}^{N}\left(\mathbb{U}\left(\mathbf{x}_{n_1,n_2}\right)\frac{\cos\left(\omega r_{n_1,n_2}\right)}{r_{n_1,n_2}}\right)^2 \\
&= \sum_{\substack{n_2\\ n_1\neq n_2}}^{N}\mathbb{U}\left(\mathbf{x}_{n_1,n_2}\right)\left(\frac{\cos\left(\omega r_{n_1,n_2}\right)}{r_{n_1,n_2}}\right)^2 \\
&\approx N\left\langle \mathbb{U}\left(\mathbf{x}\right)\frac{\cos^2\left(\omega r\right)}{r^2}\right\rangle \\
&\approx \frac{N}{R^2}
\end{aligned}
\tag{A60}
$$

It follows that (A48) remains valid after proper accommodation of secular drift of the charges.