DO PARTICLES AND ANTI-PARTICLES REALLY ANNIHILATE EACH OTHER?

Michael K.-H. Kiessling
Department of Mathematics, Rutgers University
110 Frelinghuysen Rd., Piscataway, NJ 08854, USA

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Abstract

Supported by results obtained with semi-classical quantization techniques, it is argued that positronium (Ps) may exhibit a proper quantum-mechanical ground state whose energy level lies $\approx 2m_{\rm el}c^2$ below its "hydrogenic pseudo-ground state" energy, where $m_{\rm el}$ is the empirical rest mass of the electron. While the familiar hydrogenic pseudo-ground state of Ps is caused by the electric attraction of electron and anti-electron, modified by small magnetic spin-spin and radiative QED corrections, the proper ground state of Ps is caused by the magnetic attraction of electron and anti-electron, which dominates the electric one at short distances. This finding suggests that the familiar "annihilation" of electron and anti-electron is, in reality, simply yet another transition between two atomic energy levels, with the energy difference radiated off in form of photons — except that the energy difference is huge: about 1 MeV instead of the few eV in a hydrogenic transition. In their proper ground state configuration the two particles would be so close that they would electromagnetically neutralize each other for most practical purposes, resulting in the appearance that they have been annihilated. Once in such a tightly bound state such pairs would hardly interact with normal everyday matter and would not be noticeable — except through their gravitational effects in bulk.

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1 Introduction

In this paper we will use semi-classical quantization techniques to supply theoretical evidence for the correctness of the idea that what appears to be the annihilation of a matter-antimatter pair of particles may in reality simply be a transition of the two particles into an extremely tightly bound quantum state with energy level ≈ 0, i.e. essentially $2mc^2$ below the positive energy continuum of this system. The electromagnetic signature of such a transition would be largely indistinguishable from that of an actual annihilation process. Although the two particles would not literally annihilate each other, in such an almost zero-energy ground state their charges and magnetic moments would effectively neutralize each other for most practical purposes — hence the appearance of annihilation. Similarly, what appears as the creation of a matter-antimatter particle pair would simply be the ionization of this two-particle atom from its ground state, by injecting an energy of $\approx 2mc^2$ into the system, with essentially zero total momentum, viewed from their center-of-mass (CM) frame. As a consequence, such a tightly bound state of particle and anti-particle would barely interact with normal matter particles.

If this scenario is confirmed by proper quantum-mechanical two-body computations, it would not entail the conclusion that the usual quantum field-theoretical formalism is entirely false (though its well-known divergence problems make it plain that this formalism is also not entirely satisfactory). Yet it would entail a need to modify some of it: what normally is thought of as "the vacuum" and defined accordingly would have to be replaced by a non-trivial ground state sector. The "creation and annihilation" operator formalism would have to be interpreted as describing, not the creation and annihilation of the particles themselves, but rather the transitions between quantum eigenstates; and so on.

One potentially important spinoff of our theoretical investigations is the speculation that the universe might be filled with such tightly bound matter plus anti-matter pairs in their proper atomic ground states. While individually practically invisible, altogether they may well contribute to universal gravitation (if the ground state energy is not exactly zero). In short, a significant part of the mysterious "dark matter" in the uinverse may consist of such matter-antimatter bound states. Since these tightly bound particle & anti-particle pairs would have spin 0, they are effectively bosons and may therefore provide a vindication of the bosonic dark matter models proposed already in [BGR1983] and more recently by others [HOTW2017]; see also [CERN2018a, CERN2018b] for a CERN perspective. This issue will be addressed in more detail in a subsequent publication.

The rest of the paper is structured as follows:

We will demonstrate the viability of our suggestion with a semi-classical treatment of the quintessential matter-antimatter system: Positronium (Ps); see section 2. In section 3 we comment on other particle & anti-particle annihilation scenarios, which will conclude this paper.

2 Positronium

Positronium is a hydrogen-like two-body system composed of an electron and an anti-electron mutually bound to each other by their electromagnetic attraction. However, different from hydrogen, Ps in its hydrogenic ground state is not stable, having a life span of $0.125 \times 10^{-3}\,\mu$s (p-Ps), respectively of $0.14\,\mu$s (o-Ps); here, p-Ps and o-Ps mean para-Ps and ortho-Ps, respectively, referring to whether the electron and anti-electron spins add to 0 (p-Ps) or to 1 (o-Ps). The shorter-lived p-Ps can decay through emission of two γ photons, while o-Ps requires at least three photons to decay — hence its much longer life span. There is also a small energy difference between the hydrogenic p-Ps and o-Ps ground states, known as hyperfine (HF) splitting, caused mainly by their spin-spin interaction; see [KaKl1952], [Cza1999]. In this sense the hydrogenic p-Ps ground state *is* the hydrogenic ground state of Positronium.

2.1 The hydrogenic p-Ps annihilation physics

Since the usual field-theoretical formalism implements the *annihilation/creation interpretation* of Ps, this entails which quantities one seeks to compute and which ones not, and how this is being done. In particular, we note that if one assumes that electron and anti-electron really annihilate each other, it makes no sense to ask "In which state are they?" once they are gone.[1] Thus, to compute the energy of each of the two γ photons which are being emitted in opposite directions (due to momentum conservation) in the CM frame of Ps, one just has to halve the relativistic energy of the hydrogenic p-Ps ground state. The theoretical high-precision computation of this p-Ps ground state is done perturbatively, expressed as a formal series in powers of, and powers of the logarithm of, Sommerfeld's fine structure constant $\alpha_{\mathrm{S}} = \frac{e^2}{\hbar c} \approx \frac{1}{137.036}$, which serves as the dimensionless coupling constant between electron and anti-electron. A brief recap of this standard procedure follows:

I) Ignoring gravity, the vacuum is defined to have energy zero.

II) Adding a theoretically "non-interacting electron and anti-electron pair" to the vacuum (technically: replacing α_{S} by 0), the lowest energy $E_g(0)$ of such a two-particle "universe" is just the sum of their two rest energies, $E_g(0) = 2m_{\mathrm{el}}c^2$.

III) One next takes their electric Coulomb interaction $-e^2/r$ into account, obtaining an additive correction to the "free ground state" energy which, to leading order in powers of α_{S}, is given by the lowest eigenvalue of the hydrogenic Bohr spectrum with $Z = 1$ and

[1] Since the hydrogenic ground state of Ps is not stable, it is not a quantum-mechanical ground state in the proper sense (which would be stable). However, in the usual annihilation narrative, there is no lower-energy "state of Ps" — in this sense we prefer to call the hydrogenic ground state of Ps *a pseudo-ground state*.

reduced mass $\mu = m_{\rm el}/2$,

$$E_{\rm Ps}^{\rm Bohr} = \left\{ -\frac{m_{\rm el}c^2\alpha_{\rm S}^2}{4n^2} \right\}_{n\in\mathbb{N}} ; \tag{2.1}$$

recall that (2.1) can be obtained by semi-classical techniques, or by solving either the two-body Schrödinger or Pauli equation for the electron-positron system with the Coulomb interaction $-e^2/r$ as the only interaction potential. Thus, to $O(\alpha_{\rm S}^2)$ included the hydrogenic Ps ground state energy is

$$E(\alpha_{\rm S}) = 2m_{\rm el}c^2 \left(1 - \tfrac{1}{8}\alpha_{\rm S}^2 + O(\alpha_{\rm S}^4)\right). \tag{2.2}$$

IV) The $O(\alpha_{\rm S}^4)$ corrections are obtained by adding the quantum mechanical expected values, taken with respect to the ground state wave function found by solving the two-body Pauli equation in step III, of the following operators: the relativistic kinetic energy expanded one order beyond the Newtonian term, the Darwin term, and the spin-spin coupling term — note that the spin-orbit coupling term yields no contribution in the p-Ps's $\ell = 0$-state. This yields (cf. [KaKl1952])

$$E(\alpha_{\rm S}) = 2m_{\rm el}c^2 \left(1 - \tfrac{1}{8}\alpha_{\rm S}^2 - \tfrac{5}{32}\alpha_{\rm S}^4 + O(\alpha_{\rm S}^5)\right). \tag{2.3}$$

V) The $O(\alpha_{\rm S}^5)$, and $O(\alpha_{\rm S}^5 \ln \frac{1}{\alpha_{\rm S}})$ corrections have also been computed in [KaKl1952]. By now the expansion has been pushed to $O(\alpha_{\rm S}^7 \ln^2 \frac{1}{\alpha_{\rm S}})$.[2]

For the purpose of computing the energy released through "annihilation" of the hydrogenic p-Ps ground state, one does not need such high precision calculations. Already the terms displayed in (2.3) are more than enough for this goal. Indeed, assuming that Positronium will annihilate completely when its hydrogenic p-Ps ground state decays by emitting two γ photons, it follows from energy and momentum conservation that in the CM frame each γ photon carries away an energy $E_\gamma = m_{\rm el}c^2 \left(1 - \frac{1}{8}\alpha_{\rm S}^2 - \frac{5}{32}\alpha_{\rm S}^4 + O(\alpha_{\rm S}^5)\right)$; now noting that $\frac{5}{32}\alpha_{\rm S}^4 \approx 4.43 \times 10^{-10}$, it is clear that even if all $O(\alpha_{\rm S}^4)$ terms are ignored one still achieves a relative precision of 8 decimal places.

2.2 Critique of the perturbative treatment

In the calculation of the Ps pseudo-ground state energy described above, only the electrostatic Coulomb attraction between a point electron and a point anti-electron has been

[2] These very high precision calculations are needed to match the very precise experimental measurements of the spectral line of the o-PS to p-Ps hyperfine transition which, because of cancellations of the lower order terms in the expansion of the hydrogenic o-Ps and p-Ps ground state energies, releases the energy $\frac{7}{12}\alpha_{\rm S}^4 m_{\rm el}c^2$, corrected by higher order terms; see, e.g., [Cza1999].

handled non-perturbatively,[3] while all $O(\alpha^4)$ terms, in particular, the magnetic spin-spin interaction (recall that the spin-orbit interaction does not contribute in the $\ell = 0$ state) has been treated merely perturbatively. The conventional rationale for this procedure is the following quasi-classical rule-of-thumb estimate: in the Pauli ground state of Ps with purely Coulombic interaction the two particles are most likely a distance $2\frac{\hbar}{m_{\rm el}c}\frac{1}{\alpha_{\rm S}}$ (the "Bohr radius of Positronium") apart. At this distance, the electric interaction energy is $-\frac{1}{2}\alpha_{\rm S}^2 m_{\rm el}c^2$ while the attractive magnetic dipole-dipole interaction energy attributed to the two spins, taking the Bohr magneton $\frac{1}{2}\frac{e\hbar}{m_{\rm el}c}$ as magnetic dipole strength, is a factor $\alpha_{\rm S}^2$ smaller (ignoring numerical factors of order 1) — thus a perturbative calculation should suffice to account for the influence of spin onto the hydrogenic ground state of Ps.

Our point of departure is the difference in the distance scaling laws of the electric monopole-monopole and magnetic dipole-dipole interactions. While the electric monopole-monopole interaction energy scales $\propto -1/r$, the magnetic dipole-dipole one scales $\propto -1/r^3$. Of course, the two types of interaction energy terms also have different coupling parameters, but their ratio is dimensionless and determines a unique distance at which their ratio equals unity. And so, since a perturbative treatment of the spin-spin coupling is justified *only* if the electric pair energy dominates the magnetic one, the validity of perturbative QED calculations is restricted to the far side of the break-even distance, as the electric monopole-monopole interaction weakens much more slowly than the attractive magnetic dipole-dipole interaction when the particle distance r increases. Short of the break-even distance, the situation is precisely the other way round!

The upshot is: there might be magnetically bound states of Positronium with energies way below the hydrogenic ground state energy, whose computation was outlined in the previous subsection. Their computation requires a non-perturbative treatment of the magnetic interactions of electron and anti-electron. Our calculations are non-perturbative and show the viability of a magnetically bound ground state of Ps near zero energy.

2.3 A non-perturbative semi-classical treatment of p-Ps

A definitive non-perturbative investigation of Positronium cannot be carried out by summing Feynman diagrams, and "non-perturbative QED" does not yet exist because the renormalization flow to remove its ultraviolet cutoffs has not been completed; many experts actually believe that the non-perturbative removal of the UV cutoffs is *impossible*. In such a situation the best one can do is to resort to reasonable approximations, using techniques which in the past have been employed successfully already.

In this vein, we will work with Bohr–Sommerfeld type calculations of the energy spectrum. We recall the remarkable exact coincidence of the Dirac *Hydrogen energy eigenvalue*

[3] Since the $O(\alpha_{\rm S}^2)$ energy term in (2.3) is just a small correction to the rest energy term $2m_{\rm el}c^2$, it has the appearance of being just a perturbation to the rest energy term. However, it cannot be computed perturbatively.

spectrum [Dar1928] with Sommerfeld's fine structure formula [Som1916]. Both are calculated in the Born–Oppenheimer approximation, i.e. with the proton assumed fixed, but since Dirac's model incorporates electron spin while Sommerfeld's does not, it is clear that the accounting of angular momentum differs in the two models. The special circumstances of why nevertheless the two energy spectra agree have been explained in detail in the beautiful work of Keppeler [Kep2003], who analysed the semi-classical limit of the Dirac equation for Hydrogen, and discovered a generalization of the Einstein–Brillouin–Keller quantization rules to spinning particle motions. The exact coincidence of the spectra is an exception and not the rule, but Keppeler's work explains this exception, and also the (smaller) discrepancies in the more general situations. Since our goal is not high-precision computations but a qualitative — yet mathematically compelling — demonstration that a magnetically tightly bound Positronium quantum state is possible, a Bohr–Sommerfeld treatment should be good enough for the purpose.

To get a feeling for the accuracy that can be expected from a Bohr–Sommerfeld calculation of the Positronium energy spectrum, we first compute the hydrogenic spectrum without magnetic interactions. Since we are not trying to compute the fine structure, in all our calculations we simply consider the circular Bohr orbits rather than the Darwin–Sommerfeld rosetta orbits.

Then we add the magnetic interactions of two point dipoles and find the first indication for the size of a tightly magnetically bound state with ≈ 0 energy — note that the Ps Hamiltonian with a magnetic interaction $\propto -1/r^3$ is unbounded below (which is the case for the classical as well as for the Pauli Hamiltonian; a Dirac Hamiltonian is of course unbounded below even without interactions, but that is a different issue), and the size estimate comes from where the energy function has its radial zero.

To avoid unboundedness below, the only way out is to assume that the electron and anti-electron are not true points but have a nontrivial geometric structure. We will follow Max Born (see [Rao1936]) and assume the particles are tiny rings which carry charge and current. However, different from Born's suggestion that the electromagnetic fields are to be computed with the help of the Born–Infeld modification of the Maxwell–Lorentz equations (which nobody has been able to do for ring sources so far), we are using, first, the conventional Maxwell–Lorentz equations (but then have to subtract the infinite self-energies), and second, the Bopp [Bop1940], Landé–Thomas [LaTh1941], Podolsky [Pod1942] modification of the Maxwell–Lorentz field equations, a pre-cursor to Feynman's cutoff [Fey1948], which yield finite self-energies and a finite magnetic flux through a ring.

A very subtle issue is the size of the magnetic moment assigned to the ring particles. It would seem obvious that the magnitude of the magnetic moment should be the Bohr magneton. However, as insisted on by Pauli, *the Bohr magneton* is associated with the electron spin which in turn is implemented in the Pauli and Dirac equations for point particles through the Pauli σ matrices acting on the spinors, and so *has nothing to do with any electron structure*. Any magnetic moment associated with a structure of the electron therefore has to come *in addition* to the Bohr magneton. As argued recently also

in [KTZ2016], it is very suggestive, therefore, to equip the ring structure of Born's electron with the electron's anomalous magnetic moment.[4]

To keep the formulas simpler, we will from now on use the following units: the energy unit is $m_{\rm el}c^2$, and that of mass is $m_{\rm el}$; the unit of momentum is $m_{\rm el}c$; the unit of angular momentum is $\hbar$; the unit of length is $\hbar/m_{\rm el}c$; unit of electric charge is e.

2.3.1 Two point particles with electric charges

In the CM frame of a mutually circulating electron and anti-electron, and expressed in the units just stipulated, their classical energy function reads

$$H(r,p) := 2\sqrt{1+p^2} - \tfrac{\alpha_{\rm S}}{r}, \tag{2.4}$$

with p and r constants. To find the principal Bohr–Sommerfeld energy values of Positronium one needs to minimize (2.4) subjected to Bohr's quantization condition $pr = n$ with $n = 1$; here, p is the magnitude of the electron's or anti-electron's momentum, both assumed to move in a circular path around a common center, r is their distance, and $n \in \mathbb{N}$ is Bohr's orbital angular quantum number.[5] Thus, to compute the principal Bohr–Sommerfeld energy values of Positronium one needs to minimize

$$V_n^{(1)}(r) := 2\sqrt{1+\tfrac{n^2}{r^2}} - \tfrac{\alpha_{\rm S}}{r} \tag{2.5}$$

with respect to r. (See Fig.1 for $V_n^{(1)}(r)$ when $n = 1$ (green) and $n = 2$ (blue).)

Minimization of $V_n^{(1)}(r)$, for any $n \in \mathbb{N}$, yields the energy spectrum

$$E_n(\alpha_{\rm S}) = 2\sqrt{1-\tfrac{\alpha_{\rm S}^2}{4n^2}}. \tag{2.6}$$

Maclaurin expansion in powers of $\alpha_{\rm S}$ yields

$$E_n(\alpha_{\rm S}) = 2\left(1 - \tfrac{1}{8n^2}\alpha_{\rm S}^2 - \tfrac{1}{128n^4}\alpha_{\rm S}^4 + O(\alpha_{\rm S}^6)\right). \tag{2.7}$$

The ground state energy of Coulombic Ps using Bohr–Sommerfeld rules is obtained by setting $n = 1$ in (2.1); note that the $O(\alpha_{\rm S}^4)$ term in (2.7) differs from the correct term (in absence of spin-spin interactions) by a factor 4, so this calculation is correct up to terms of $O(\alpha_{\rm S}^2)$ included, but it only gives the tendency (i.e. the sign) of the $O(\alpha_{\rm S}^4)$ term correctly.

[4]There is another logical possibility: any structure of the electron makes yet another contribution to the "magnetic moment of the electron," in addition to the Bohr magneton size moment associated with the spinor structure of Pauli and Dirac equations, and in addition to the anomalous magnetic moment computed perturbatively using QED. In this article we are not pursuing this option but leave it to some future investigation.

[5]Of course, one knows since 1925 that n is not the correct quantum-mechanical orbital angular momentum quantum number, but this is immaterial for the computation of the energy levels.

2.3.2 Two point particles with electric charges and magnetic dipoles

The anomalous magnetic moment of the particles, itself computed perturbatively from QED, yields a correction to the spin-orbit coupling already inherent in Dirac's equation, plus a magnetic dipole-dipole type interaction energy which is itself treated with first-order perturbation theory, each yielding a tiny correction to the Coulombic spectrum. Here we add an attractive anomalous magnetic dipole-dipole interaction in an ad-hoc manner to (2.4), implement Bohr's quantization condition, and look for (local) minima of

$$V_n^{(2)}(r) := 2\sqrt{1+\frac{n^2}{r^2}} - \frac{\alpha_S}{r} - \frac{1}{8\pi^2}\frac{\alpha_S^3}{r^3} \tag{2.8}$$

with respect to r, for $n = 1$, see Figs.1 and 2.

Fig.1 shows the usual hydrogenic regime which is dominated by the Coulomb interaction of the two point charges. Both $V_n^{(1)}(r)$ and $V_n^{(2)}(r)$ are plotted versus r, each for $n = 1$ (lower curves) and $n = 2$ (upper curves). Recall that the unit for V_n is $m_{\rm el}c^2$, the unit for r is $\frac{\hbar}{m_{\rm el}c}$. The local minimum for $n = 1$ occurs for r roughly equal to the "Bohr radius of Positronium," $2\frac{\hbar}{m_{\rm el}c}\frac{1}{\alpha_S}$, i.e. $r \approx 274$ in our electron Compton length units. For each n value the two pertinent curves are virtually indistinguishable at this leading order energy scale, confirming that a perturbative treatment of magnetic effects is justified in this regime.

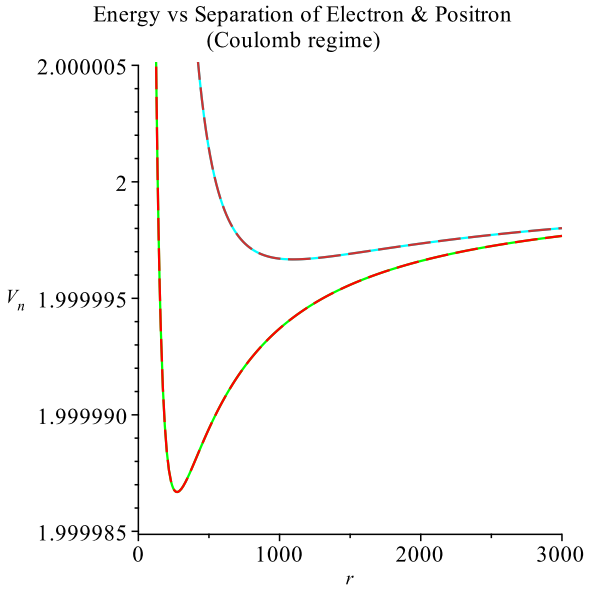

Figure 1

Not visible on the scale shown in Fig.1 is the behavior of both $V_n^{(1)}(r)$ and $V_n^{(2)}(r)$ for

separations r very much smaller than the "Bohr radius of Positronium." This very different scale is shown in Fig.2, now only for $n = 1$; the color code is the same as in Fig.1.

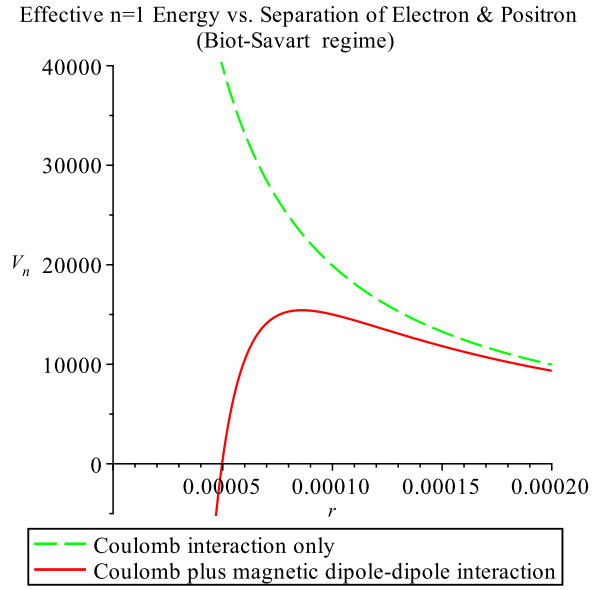

Figure 2

Figure 2 reveals that a tightly bound state due to the magnetic dipole-dipole type attraction of electron and positron is feasible, provided at very short distances the $-1/r^3$ behavior is mollified into something gentler, diverging to $-\infty$ not faster than $-1/r$ because the relativistic angular momentum barrier scales $\propto 1/r$ for small r. The transition from $-1/r^3$ scaling to something milder has to happen for distances smaller than 10^{-4} reduced Compton lengths of the electron. This is accomplished by assuming the electron (and the positron) are little rings, an idea which goes back to Max Born it seems.

2.3.3 Two ring particles with electric charges and currents: Part I

We now assume that both electron and positron are tiny rings of radius R, carrying an electric charge $\mp e$, and an electric current $\mp I$ such that $\pi R^2 I \frac{1}{c} = \frac{\alpha_S}{2\pi} \frac{e\hbar}{2m_{el}c}$. Eliminating I this way leaves one with R free to adjust. Given the distance r between the geometrical centers of the two rings, we minimize their mutual electromagnetic energy as computed with the Maxwell–Lorentz fields. The minimum occurs if the rings are co-planar, oriented such that their interaction energy increases with r. These energies have been computed many times in the literature in terms of complete elliptic integrals **K** and **E**; see [AbSt1972].

Thus we obtain for the to-be-minimized energy function $V_n^{(3)}(r;R)$:

$$V_n^{(3)}(r;R) := 2\sqrt{1+\tfrac{n^2}{r^2}} - \tfrac{1}{\pi}\tfrac{\alpha_{\mathrm{S}}}{R}\frac{1}{\sqrt{1+\frac{r^2}{4R^2}}}\mathbf{K}\left(\frac{1}{\sqrt{1+\frac{r^2}{4R^2}}}\right) \tag{2.9}$$
$$- \tfrac{1}{4\pi^3}\tfrac{\alpha_{\mathrm{S}}^3}{R^3}\sqrt{1+\tfrac{r^2}{4R^2}}\left[\left(2-\frac{1}{1+\frac{r^2}{4R^2}}\right)\mathbf{K}\left(\frac{1}{\sqrt{1+\frac{r^2}{4R^2}}}\right) - 2\mathbf{E}\left(\frac{1}{\sqrt{1+\frac{r^2}{4R^2}}}\right)\right]$$

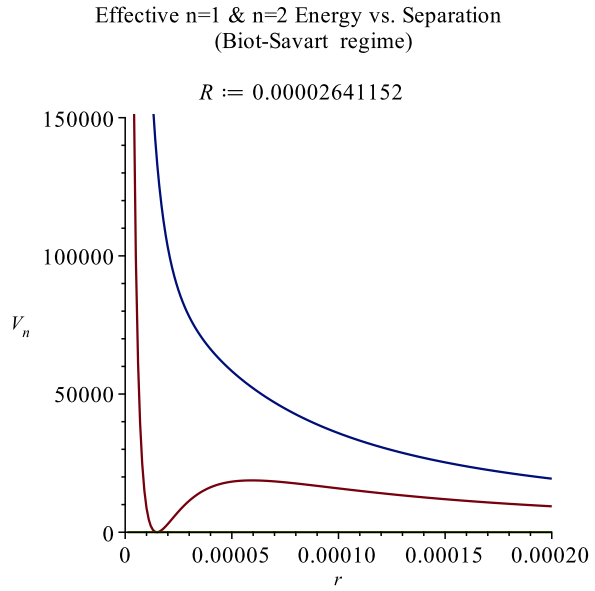

Figure 3

In Fig.3 we show $V_n^{(3)}(r;R)$ for $n=1$ (lower curve) and $n=2$ (upper curve) in the Biot–Savart dominated regime when $R = 0.4959784\alpha_{\mathrm{S}}^2$. We see a global minimum of the $n=1$ curve for $r \approx 1.3\times 10^{-5}$ reduced electron Compton lengths that corresponds to a tightly bound $n=1$ state with energy ≈ 0, indeed; we also see that for $n=2$ there is no additional tightly bound state (in fact, this is true for any $n>1$). We also remark that in the Coulomb dominated regime (not shown) the energy curves are virtually indistinguishable from those shown in Fig.1, producing the hydrogenic Bohr spectrum, as they should.

2.3.4 Two ring particles with electric charges and currents: Part II

We continue to assume that both electron and positron are tiny rings of radius R, carrying an electric charge $\mp e$, and an electric current $\mp I$ such that $\pi R^2 I\frac{1}{c} = \frac{\alpha_{\mathrm{S}}}{2\pi}\frac{e\hbar}{2m_{\mathrm{el}}c}$. As before, eliminating I this way leaves one with R free to adjust.

However, to demonstrate the robustness of a tightly magnetically bound Ps state, we now compute the interaction energies with the Bopp–Landé–Thomas–Podolsky modification of the Maxwell–Lorentz field equations, see [Bop1940], [LaTh1941], [Pod1942] for the original works, and [KTZ2018b] for a modern rigorous assessment of its classical merits.

In the MBLTP field theory the $\boldsymbol{\mathcal{H}}$ and $\boldsymbol{\mathcal{D}}$ fields are the same as in the Maxwell–Lorentz theory, but the law of the electromagnetic vacuum of the ML theory, $\boldsymbol{\mathcal{H}} = \boldsymbol{\mathcal{B}}$ and $\boldsymbol{\mathcal{D}} = \boldsymbol{\mathcal{E}}$, is replaced by partial differential equations which in the static limit reduce to

$$\boldsymbol{\mathcal{H}}(\boldsymbol{s}) = \big(1 - \varkappa^{-2}\Delta\big)\,\boldsymbol{\mathcal{B}}(\boldsymbol{s})\,, \tag{2.10}$$

$$\boldsymbol{\mathcal{D}}(\boldsymbol{s}) = \big(1 - \varkappa^{-2}\Delta\big)\,\boldsymbol{\mathcal{E}}(\boldsymbol{s})\,; \tag{2.11}$$

here, Δ is the Laplacian, and $\varkappa$ is Bopp's fundamental inverse length. The parameter $\varkappa$ is undetermined as of yet; however, in [CKP2018] the Hydrogen spectrum was studied and it was found that the currently available precision of the Lyman α fine structure sets a lower limit of $\approx \frac{1}{2} \times 10^{18}\mathrm{m}^{-1}$, or $\approx 2 \times 10^5$ reciprocal reduced Compton length units, for $\varkappa$.

While the solutions of the Maxwell–Lorentz field equations for a point or ring electron have an infinite total field energy, the solutions of the Maxwell–Bopp–Landé–Thomas–Podolsky field equations have a finite field energy — a welcome feature of this theory! The interaction energy between two ring particles carrying electric charges and currents is now defined as usual as the total field energy of such a configuration minus the self-field energies of its constituents, but unlike for Maxwell–Lorentz fields, this does not now amount to an uncomfortable "infinite self-energy subtraction." In the static limit (relevant to computing the interaction energy needed for the determination of quantum mechanical energy spectra) the total electromagnetic energy of the MBLTP fields is given by the familiar expression[6]

$$E_{\text{field}} = \frac{1}{8\pi}\int_{\mathbb{R}^3} \left(\boldsymbol{\mathcal{E}}\cdot\boldsymbol{\mathcal{D}} + \boldsymbol{\mathcal{B}}\cdot\boldsymbol{\mathcal{H}}\right) d^3s \geqslant 0. \tag{2.12}$$

For the ring particles we are discussing right now this is easily reduced to one-dimensional quadratures which, to the best of our knowledge, have not been expressed in terms of already known special functions (such as complete elliptic integrals in the Maxwell–Lorentz case, for instance). Thus we now have to minimize

$$\begin{aligned} V_n^{(4)}(r;R,\varkappa) := 2\sqrt{1+\tfrac{n^2}{r^2}} &- \tfrac{\alpha_{\mathrm{S}}}{2\pi R}\int_0^\pi \frac{1-\exp\left(-2\varkappa R\sqrt{\sin^2(\varphi)+\frac{r^2}{4R^2}}\right)}{\sqrt{\sin^2(\varphi)+\frac{r^2}{4R^2}}}\mathrm{d}\varphi \\ &- \left(\tfrac{\alpha_{\mathrm{S}}}{2\pi R}\right)^3\int_0^\pi \cos(2\varphi)\frac{1-\exp\left(-2\varkappa R\sqrt{\sin^2(\varphi)+\frac{r^2}{4R^2}}\right)}{\sqrt{\sin^2(\varphi)+\frac{r^2}{4R^2}}}\mathrm{d}\varphi \end{aligned} \tag{2.13}$$

w.r.t. r, and with R and $\varkappa$ to be determined wisely — by this we mean the following.

Since the BLTP vacuum law of electromagnetism not only renders the field energies etc. finite, but also the magnetic flux through a ring particle which carries an electric charge and current (neither is true for the fields computed with the ML vacuum law), we impose

[6] The non-negativity of the integral can be shown with the help of the BLTP vacuum law, (2.10) expressing $\boldsymbol{\mathcal{H}}$ in terms of $\boldsymbol{\mathcal{B}}$, and (2.11) expressing $\boldsymbol{\mathcal{D}}$ in terms of $\boldsymbol{\mathcal{E}}$, followed by integration by parts. We remark that (2.12) is valid only in static situtations.

a heuristic relationship on $\varkappa$ and R by demanding that the magnetic self-field flux through a ring equals the empircal magnetic flux quantum, $e\frac{1}{\alpha_S}$. This yields the constraint

$$R = \frac{\alpha_S^2}{2\pi}\int_0^\pi \cos(2\varphi)\frac{1-\exp\left(-2\varkappa R\sin(\varphi)\right)}{\sin(\varphi)}\mathrm{d}\varphi. \tag{2.14}$$

With (2.14) in place we have adjusted the product $\varkappa R$ until the lowest energy minimum was essentially zero. This has yielded $R \approx 2.57 \times 10^{-5}$ and $\varkappa \approx 1.8 \times 10^5$.

In Fig.4 we show $V_n^{(4)}(r; R, \varkappa)$ (solid lines) for $\varkappa \approx 1.8 \times 10^5$ and $R \approx 2.57 \times 10^{-5}$, both for $n = 1$ and $n = 2$, together with a truncated version of $V_n^{(4)}(r; R, \varkappa)$ which omits the magnetic interactions, retaining only the electric interaction (dashed lines). The figure shows precisely one bound state in the Biot–Savart regime, associated with the $V_1^{(4)}(r; R, \varkappa)$ curve, while the $V_n^{(4)}(r; R, \varkappa)$ curves for $n > 1$ do not have a local minimum in the Biot–Savart regime — they all do have a global minimum in the Coulomb regime, of course, where the $n = 1$ curve has a local minimum in addition to the global one shown in Fig.4.

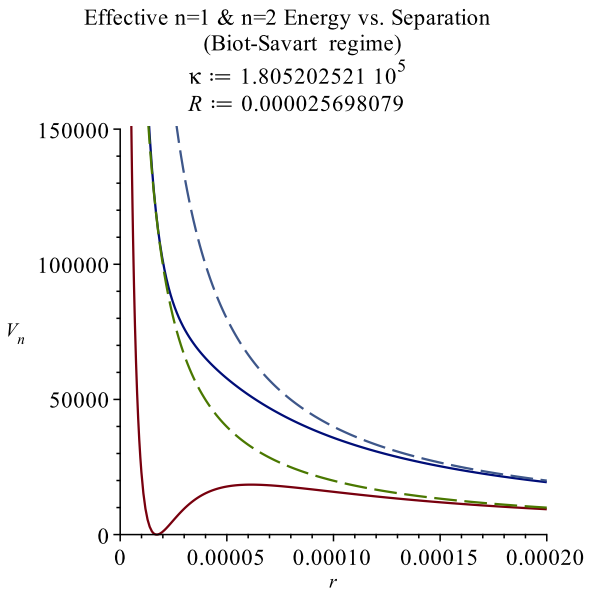

Figure 4

In Fig.5 we show a blow-up (or zoom-in) of the $V_1^{(4)}(r; R, \varkappa)$ curve, with $\varkappa \approx 1.8 \times 10^5$ and $R \approx 2.57 \times 10^{-5}$. It is obvious that the ground state energy is practically zero.

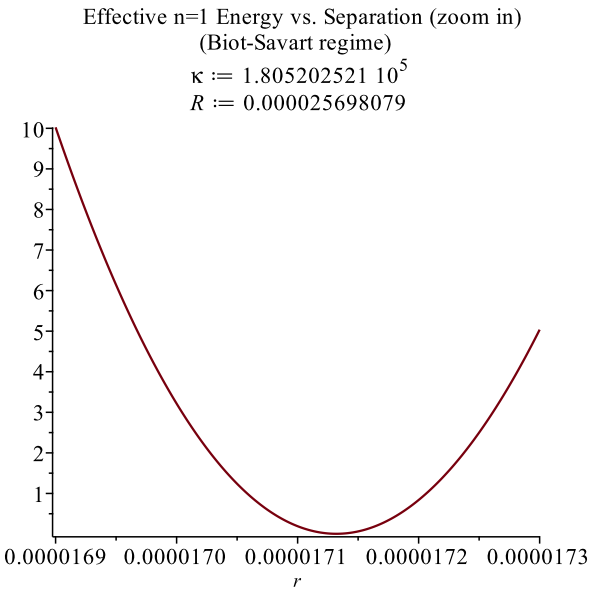

Figure 5

As before, the continuation of the energy curves to the Coulomb dominated regime (not shown) reveals that there they are virtually indistinguishable from those shown in Fig.1, producing the hydrogenic Bohr spectrum, as they should.

Although the fixing of $\varkappa$ and R is admittedly heuristical, it is perfectly reasonable. What is comforting is that the obtained value of $\varkappa$ is in line with the lower estimate for it obtained in [CKP2018], and that the r value of the global energy minimum comes out roughly the same in all three calculations presented here. Moreover, the value of the ring radius R comes out about the same in the two ring model calculations presented here. This demonstrates that the existence of a magnetically bound ground state of Positronium with energy near zero is a viable conjecture.

3 Summary and Outlook

In this paper we have investigated the bound states of an electron and an anti-electron using the Bohr–Sommerfeld quantization techniques which appeared at the dawn of quantum physics. While these may seem like a pre-enlightenment approach to quantum physics, one should not forget that the Hydrogen energy fine structure spectrum came out exactly right by these calculations, so that the here computed Positronium spectrum may not be too far from the truth either. The conclusion is that the magnetic binding of electron and anti-electron may be so strong as to cause a true ground state of the two particles near zero total energy. What is normally interpreteted as the annihilation of the two particles

may therefore simply be a transition from one of the low-lying hydrogenic (i.e. Coulomb dominated) energy levels to the magnetic (i.e. Biot–Savart dominated) ground state level, the electromagnetic signature of which being virtually indistinguishable from an actual annihilation of the two particles.

If confirmed by more advanced quantum-mechanical calculations, in particular for two zGKN type rings (see [KTZ2016]), this novel ground state may mean that the universe is full of small mass, effectively bosonic particles which could form a large part of the mysterious dark matter in the universe. We will address this issue in more detail in a follow-up publication. Here we comment on the generalization of our ideas to other matter & anti-matter systems.

It does not take too much imagination to come up with the conjecture that the Ps spectrum suggested in this paper is representative of the spectra of other atomic matter & anti-matter systems, such as quarkonium etc. Those systems likewise would have to be considered contributing to the dark matter in the universe.

But what about the photon? Photons surely get created and annihilated, don't they? Not necessarily! Of course, if one literally defines a photon, as is done in QED, as "a spin-1 particle with zero rest mass and momentum $\hbar\boldsymbol{k}$," then as soon as the momentum of the photon changes from $\hbar\boldsymbol{k}$ to $\hbar\boldsymbol{k}'$, "the original $\boldsymbol{k}$ photon" is gone, and "a $\boldsymbol{k}'$ photon" has appeared. But this is not distinguishable from the scenario in which a photon is really a particle, perhaps a point particle, which simply changes its momentum from $\hbar\boldsymbol{k}$ to $\hbar\boldsymbol{k}'$, very much as envisaged by Einstein, see [KTZ2018a]. It is quite conceivable, and logically perfectly admissible, that the emission / absorption of photons by atoms does not mean that photons are created / annihilated in this process, but simply ejected / captured. In a forthcoming paper [KLTZ2018] it will be demonstrated that this scenario is indeed happening in a relativisitic photon-electron model in 1+1 spacetime dimensions.

The upshot is: at the Big Bang (if there was a true singularity) our universe may have started with a huge but finite number of particles in it, and all these are still around. We may already know what type of particles these are, all or most of them — no need for speculative exotic matter models and such! And if dark matter is indeed mostly tightly bound particle & anti-particle pairs, it would mean that the apparent total lopsidedness of matter over anti-matter in the visible universe (see [CERN2018b]) may be a misleading appearance. The universe could be filled in almost equal amounts with matter and with anti-matter, with only a tiny imbalance — but this imbalance is precisely what is visible to us.

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