The GHSZ argument: a gedankenexperiment requiring more denken

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Abstract

The wonderful gedankenexperiment of Greenberger, Horne, Shimony and Zeilinger has received only partial analysis in their seminal presentation. The analysis is completed here, yielding the consistent EPR model for which they searched. The fatal error leading to their flawed conclusion arises from the incompleteness of their analysis, an error of neglect.

1 Prelude

Please continue reading, with an open mind. The surprising results of this note defy the influential conclusions of Greenberger, Horne, Shimony and Zeilinger (1990) who expounded a version of Bell's theorem without involving inequalities. Their argument, in tandem with experimental developments culminating in the empirical work of Hensen, Bernien, Drau et al.(2015) have led Wiseman (2015) to advise the physics community of the "death by experiment for local realism" in a recent issue of *Nature*. Aware of the popularity of this conclusion, I ask you to continue nonetheless, if only out of intrigue. However, I do hope you find my analysis of the situation to be enjoyable, insightful, and convincing. I shall make some further comments in a concluding section. This edition revises minor errors in a previous submission.

I presume any reader's detailed familiarity with the work of GHSZ. I use their notation fastidiously, and I often refer to their equations by number, without reprinting them. This will spare me any further repetitive introduction. It would be best if you have pages 1134-1135 in front of you as you read this. To distinguish their equation numberings from a few that I will require for my own introduced equations here, I shall number mine with lower case Roman numerals i, ii, iii, and iv. We shall begin directly with discussion of the developments presented in their Section III which is the exclusive subject of this note. I mention only in passing that the persuasive exposition of Bell's original Theorem appearing in their Section II continues to embed an error which Bell and subsequent advocates of the supposed violation have missed. But that is another story. By Section 2.2 I will have identified a fatal error involved in the analysis of GHSZ Section III, and will have resolved it, completing the analysis in Section 2.3. Let's get right to the quick.

2 Let's go!

To begin, I request you to review Section III of GHSZ to refresh your memory of the notation and the details of their argument. I am in complete agreement with their analysis through equation (16). For clarification of a central aspect of the situation which will be relevant to all that follows, I display below the realm matrix of all measurement vectors that can possibly arise in the conduct of a specific Stern-Gerlach experiment on four entangled particles in the quantum state $|\mathbf{\Psi}\rangle$ corresponding to *any* specific experimental design for which $\phi_1 + \phi_2 - \phi_3 - \phi_4 \ = \ 0$. This condition is a supposition ("If clause") of their equation $(11a)$ which does not stand on its own without this clause. Notice that their companion equation ($11b$) relies on an alternative condition that $\phi_1 + \phi_2 - \phi_3 - \phi_4 \;=\; \pi$. Contradictory to each other, both of these conditions cannot be satisfied in any specific experimental run. Throughout this text I shall refer to these conditions as "condition ($11a$)" and "condition ($11b$)", for brevity, without repeating them in full at each instance.

In displaying the realm matrix below, I denote the specific experimental quantity measurement vector as $(A_\lambda(0), B_\lambda(0), C_\lambda(0), D_\lambda(0))^T$, signifying that each of the four angles ϕ_i is equal to 0 radians. Of course this design of the angles does satisfy condition ($11a$). This constitutes no loss of generality in representing the realm matrix for any quantity vector resulting from a design that meets this angle condition.

Since this restriction on $\phi_1+\phi_2-\phi_3-\phi_4$ implies via equation (9) of GHSZ that the expected spin product $A_\lambda(0)B_\lambda(0)C_\lambda(0)D_\lambda(0)$ equals -1, it would be impossible to achieve any experimental spin results that allow the vector of these multiplicands to imply a positive product. Thus, the realm matrix for the column vector of these four-particle measurements is

$$\mathbf{R}\begin{pmatrix} A_\lambda(0) \\ B_\lambda(0) \\ C_\lambda(0) \\ D_\lambda(0) \end{pmatrix} \;=\; \begin{pmatrix} 1 & 1 & 1 & -1 & -1 & -1 & -1 & 1 \\ 1 & 1 & -1 & 1 & -1 & -1 & 1 & -1 \\ 1 & -1 & 1 & 1 & -1 & 1 & -1 & -1 \\ -1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 \end{pmatrix} \;\equiv\; \mathbf{R}_{-1} \quad . \qquad (i)$$

In the entangled state of the four-particle system specified by $|\mathbf{\Psi}\rangle$ in equation (7), the columns of this matrix exhaust the vector values of measurements that can arise from such an experiment.

Notice the concluding definition of the denotation $\mathbf{R}_{-1}$ in equation (i). This is to distinguish it from a companion matrix I denote by $\mathbf{R}_{+1}$ which specifies the realm matrix corresponding to the possible outcomes of a different experiment in which the magnet angles satisfy instead condition ($11b$), that $\phi_1 + \phi_2 - \phi_3 - \phi_4 \;=\; \pi$. Again, without loss of generality, an exemplar experiment would generate an observable result $(A_\lambda(\pi), B_\lambda(0), C_\lambda(0), D_\lambda(0))^T$, with realm matrix

$$\mathbf{R}\begin{pmatrix} A_\lambda(\pi) \\ B_\lambda(0) \\ C_\lambda(0) \\ D_\lambda(0) \end{pmatrix} \;=\; \begin{pmatrix} 1 & -1 & -1 & -1 & 1 & 1 & 1 & -1 \\ 1 & -1 & 1 & 1 & -1 & -1 & 1 & -1 \\ 1 & 1 & -1 & 1 & -1 & 1 & -1 & -1 \\ 1 & 1 & 1 & -1 & 1 & -1 & -1 & -1 \end{pmatrix} \;\equiv\; \mathbf{R}_{+1} \quad . \qquad (ii)$$

The restrictions of the measurement vector possibilities embedded in the realm matrices $\mathbf{R}_{-1}$ and $\mathbf{R}_{+1}$ derive from equation (9) of GHSZ, which specifies that $E^\psi(\hat{\mathbf{n}}_1, \hat{\mathbf{n}}_2, \hat{\mathbf{n}}_3, \hat{\mathbf{n}}_4) \;=\; -cos(\phi_1 + \phi_2 - \phi_3 - \phi_4)$. At the two extreme angle restrictions we have entertained, that $\phi_1 + \phi_2 - \phi_3 - \phi_4$ equal 0 or π, this negative cosine value equals -1 and $+1$ respectively. This is what restricts the measurement realms to be $\mathbf{R}_{-1}$ and $\mathbf{R}_{+1}$ in these extreme cases. If the combination of experiment angles in this equation equals some other value, say, $\theta \in (0,\ \pi)$, then the realm matrix of possibilities for the measurements of the four electron spins would be the concatenation of these two realms, $[\mathbf{R}_{-1}\ \mathbf{R}_{+1}]$. With the recognition of this situation clearly in mind, we are ready to face the wall.

2.1 Hitting a wall at equation (16)

GHSZ regard their conclusion (16), that $A_\lambda(2\phi) = A_\lambda(0) = \text{const}$ for any angle ϕ, as surprising, for reasons they well explain. Recognizing that nonetheless this equation is not mathematically contradictory in itself, they pursue a contradiction by recourse to their equation (11b) which had not yet been brought into play in the derivation of (16).

We shall indeed join GHSZ in this pursuit, but we should recognise at the outset that the results of (11a) and (11b) pertain to contradictory experimental designs of the Gerlach-Stern magnet settings. The former result pertains to settings for which $\phi_1 + \phi_2 - \phi_3 - \phi_4$ equals 0, while the latter pertains when $\phi_1 + \phi_2 - \phi_3 - \phi_4$ equals π. Enough said for now, but note that it is contradictory and thus impossible for *both* of these conditions to hold in any experiment on any quartet of particles. Wondering ourselves now, how might the angle ϕ_1 at station A equal both 2ϕ and equal 0 so as to instantiate equation (16)? What could such notation mean?

2.1.1 Filling out the notation

GHSZ might well have first thought a bit further about the consequences and meaning of equation (16). To begin, we ought recognize this equation to imply that *all* measurements $A(\phi)$ need equal $A(0)$ for every angle ϕ under design conditions pertinent to (11a). It is irrelevant whether one refers to an arbitrary angle as ϕ or as 2ϕ. Recall that in the shorthand notation of GHSZ being used here, $A(0)$ initially represents the value of the measurement A *when all four* design angles equal 0, as per equation (12a) where it first appears. This equation denotes that the directions of all four Stern Gerlach magnet orientations displayed in Figure 2 are identically aligned with the x-axis.

Now it may sound surprising as a boldly stated result, to hear further that the physical symmetry of the experimental setup also requires that $B(\phi) = B(0)$, that $C(\phi) = C(0)$, and that $D(\phi) = D(0)$ for every angle ϕ as well! (Note that each of these equalities must be understood in the context that both sides of the equations are pertinent to designs in which $\phi_1 + \phi_2 - \phi_3 - \phi_4$ equal 0. An exemplar setting would be that each angle ϕ_i equals the same arbitrary angle ϕ.) However, these results are not really surprising either. There is surely nothing special about the component A of the four measurements playing the role that it does in the derivation of (16). Algebraically, the angles associated with the A's in derivational lines of equations (12) could be permuted with those of the B's to achieve the corresponding equalities. Then if the angles A and B in these lines were permuted with those in the lines of C and D, we would again achieve the corresponding equalities for the C's and the D's, even allowing permutation of the C's and D's. For geometrical motivation, one could merely imagine flipping the image in Figure 2 by 180° in the y-direction and/or in the z-direction, relabeling the locations A, B, C, and D appropriately.

Thus, we can understand $A(\phi)$ in equation (16) to represent the measurement value of A in an experiment in which *all four of the angles* ϕ_1, ϕ_2, ϕ_3 and ϕ_4 are identically equal to an arbitrary angle ϕ. That is why the numerical subscripts are removed from ϕ.

2.1.2 Recognizing rotational symmetry

Understood in this way, the stated conclusion (16) which we write as $A(\phi) = A(0)$ for all ϕ can be expanded to say $(A_\lambda(0), B_\lambda(0), C_\lambda(0), D_\lambda(0)) \ = \ (A_\lambda(\phi), B_\lambda(\phi), C_\lambda(\phi), D_\lambda(\phi))$. In this fully expressive form, this is evidently a condition of rotational symmetry. Identical direction vectors $\hat{\mathbf{n}}_i$ at the four observation stations determining a common angle $\phi_i = \phi$ can be rotated together in the (x,y) plane of the experiment however one might wish, without changing the experimental result. Importantly, notice that this is not merely a restriction on the expectation of the spin products at an equi-angular experiment. We knew this before the derivation of equation (16) began in equations $(12a - 12d)$. Result (16) is a condition on the actual results of an experiment on a specific set of particles designed to meet the angle condition (11a). The actual result on a specific quartet of particles must be invariant with respect to the rotation of the (x, y) axis.

In the context of the locality presumption of EPR, this restriction of all equal angles in the understanding of $A(\phi)$ and $A(0)$ shall be found unduly severe, but it is worth considering this situation to begin this discussion. What is relevant, and continues to be relevant to understanding this conclusion of symmetry, is that $A(\phi)$ need equal $A(0)$ whenever the angle $\phi_1 = \phi$

or $\phi_1 = 0$ *in the contexts of specified experimental designs for which the designed magnet angle combination* $\phi_1 + \phi_2 - \phi_3 - \phi_4$ *equals* 0 . Some further (but not all) instances of this more general understanding of the notation appear in equations ($12b, 12c, 12d$).

With this recognition of its shorthand expressiveness, equation (16) is not surprising at all, on account of the rotational and permutation symmetries of the experiment as designed! Algebraically, several symmetries are evident in the QM-motivated stipulations of equations (8) and (9), which show that the expected 4-spin-product is a function only of the angle combination $\phi_1 + \phi_2 - \phi_3 - \phi_4$. Rotational symmetry of the experimental conditions would be exhibited in the transformation of the vector of angles $\mathbf{\Phi}_4$ by the addition of any vector of constants $\mathbf{t}_4 = (t, t, t, t)$, which would surely preserve this angle combination. In fact, preservation would continue under the addition of any angle vector $\mathbf{t}_4$ for which $t_1 + t_2 = t_3 + t_4$. Furthermore, permutations of either or both of ϕ_1 with ϕ_2 and/or ϕ_3 with ϕ_4 in the specification of $\mathbf{\Phi}_4$ would preserve this condition as well. Geometrically, the former would be exhibited by rotating the (x, y) planes containing the directional vectors $(\hat{\mathbf{n}}_1, \hat{\mathbf{n}}_2, \hat{\mathbf{n}}_3, \hat{\mathbf{n}}_4)$ in Figure 2 around the orthogonal $-z \leftrightarrow z$ axes, all at the same rate. Permutation symmetry would be exhibited by flipping the z-axis systems of either angular pair (ϕ_1, ϕ_2) or (ϕ_3, ϕ_4) at the source node shown in Figure 2.

2.1.3 The bottom line on equation (16)

All this means is that if one were to evaluate the four spin measurements of a single instance of the four entangled electrons (finding the measurements to equal, say, 1, -1, 1, 1) at any angles $\mathbf{\Phi}_4$ whatsoever that satisfy the conditions of ($11a$) the measurements would be the same as they would if the angles were transformed to another configuration that satisfies the same conditions! Be clear on what this means. It does not mean that all the four spin measurements are equal. (In the case of $\phi_1 + \phi_2 - \phi_3 - \phi_4 \; = \; 0$ which we are considering, this would indeed be impossible.) However, suppose as a special case, that one perform the experiment with the orientations of the various axes as depicted in Figure 2, and with all four angles designed to equal 0. This means all four of the directional vectors $\hat{\mathbf{n}}_i$ would align with the x-axis in their respective quadrants. Then observing the results, say, $A_\lambda(0) = 1, B_\lambda(0) = -1, C_\lambda(0) = 1,$ and $D_\lambda(0) = 1$, one could be certain (in the context of the EPR paradigm) that the same results would be achieved *using these same four electrons* if the experimental setup had been flipped or rotated into any one of the rotated or flipped directions we have just considered. Conducting such a transformed experiment would be equivalent to changing the perspective from which we are viewing the same experiment.

As a companion clarification, notice too that the same result of $(1, -1, 1, 1)$ would *not* necessarily be observed if we conducted the experiment on *a different quartet of electrons* prepared in the same way at any one of the orientations of the setup considered. The result of this subsequent and distinct experiment could yield as the measurement vector any one of the columns of the realm matrix $\mathbf{R}_{-1}$. This observational result too, whatever it may be, would remain constant with respect to rotational and permutation transformations of the apparatus.

Up to this point, I believe everyone who concurs with the GHSZ analysis through equation (16), including me, should be in complete agreement. All that my discussion till now has done is highlight some further implications of the analysis which has led to what they consider to be their surprising conclusion (16). I did at first share their surprise, for the good reasons they mention: "For if $A_\lambda(\phi)$ is intended, as EPR's program suggests, to represent an intrinsic spin quantity, the values of $A_\lambda(0)$ and $A_\lambda(\pi)$ would be expected to have opposite signs." (They were alluding to an instantiation of (16) in which $\phi = \pi/2$.) We have now learned, to the contrary, these measurement are required to be equal instead! Something needs to give. [As a hint for your intuiting the resolution, remember the condition on this equality of $A_\lambda(0)$ with $A_\lambda(\pi)$: the angles 0 and π in the arguments on the two sides of this equality must each be presumed to be embedded in a configuration of the angle vector $\mathbf{\Phi}_4$ that satisfies the condition of ($11a$) .]

2.2 Pushing on to a resolution of the impasse ! What is "the contradiction" ?

GHSZ continue: "The trouble becomes manifest, and an actual contradiction emerges, when we use (11b) — which until now has not been brought into play — to obtain

$$A_\lambda(\theta+\pi)B_\lambda(0)C_\lambda(\theta)D_\lambda(0) \;=\; 1 \quad , \qquad (17)$$

which in combination with Eq. (12b) yields

$$A_\lambda(\theta+\pi) \;=\; -A_\lambda(\theta) \quad . \qquad (18) \text{ "}$$

Let's think about our situation some more. The locality presumption of EPR stipulates that the value of any specific measurement, such as $A_\lambda(\theta+\pi)$ should result, for a specific experimental setup on a quartet of electrons prepared in the entangled state $|\mathbf{\Psi}>$, in a numerical outcome that does not depend on the other three angles concurrent in the experiment. In combination with the symmetry results we have just derived from (11*a*), viz., $A_\lambda(\theta+\pi) = A_\lambda(0)$ and $C_\lambda(\theta) = C_\lambda(0)$, this would seem to imply straightforwardly that

$$A_\lambda(\theta+\pi)B_\lambda(0)C_\lambda(\theta)D_\lambda(0) \;= A_\lambda(0)B_\lambda(0)C_\lambda(0)D_\lambda(0) \;=\; -1 \;\;,\; \textit{not} \;+1 \;\; ! \qquad (iii)$$

In light of GHSZ equation (17), this would indeed be a surprise! What is happening here?

We could merely remark that equation (12*b*) holds only because it satisfies the condition (11*a*) as opposed to (11*b*) which contradicts it. Presuming that both (11*b*) *and* (12*b*) hold at the same time would embed a contradiction into the argument, easily yielding nonsensical results. However, we might profitably discuss the situation further.

Remember, our understanding of the results derived from (11*a*), viz., $A_\lambda(\theta+\pi) = A_\lambda(0)$ and $C_\lambda(\theta) = C_\lambda(0)$, presumes the denotations $A_\lambda(\theta+\pi)$ and $C(\theta)$ pertain to the outcome of an experiment in which $\phi_1 = (\theta+\pi)$ *and* $\phi_3 = \theta$ *in the context of four angles* $\mathbf{\Phi}_4$ *satisfying* (11*a*). With this understood, it would be a direct application of our symmetry result to write

$$A_\lambda(\theta+\pi)B_\lambda(0)C_\lambda(\theta)D_\lambda(\pi) \;= A_\lambda(0)B_\lambda(0)C_\lambda(0)D_\lambda(0) \;=\; -1 \quad . \qquad (iv)$$

Notice the difference in the arguments of the multiplicand D's in the left-hand-side products of equations (*iii*) and (*iv*). Equation (*iii*) might then be thought to hold on account of *(iv)* via the mechanical substitution of $D_\lambda(0)$ for $D_\lambda(\pi)$ in (*iv*), using the result that $D_\lambda(\pi) = D_\lambda(0)$ under condition (11*a*). However, making *this* substitution *alone* without further accompanying substitutions on angles $\phi_1 = (\theta+\pi)$ and $\phi_3 = \theta$ to ensure the overt subscription of the angles' settings to the conditions of (11*a*) would necessitate a very contorted understanding of the left-hand side of (*iii*).

2.2.1 On shorthand notation

One need be very careful in the use of shorthand notation. The denotation $A(\phi)$ *could* be meaningful in itself. It could represent merely the numerical result of an experiment on a quartet of photons in which the angle $\phi_1 = \phi$. However, when the notation $A(\phi)$ is defined casually as such a designation in the context of a non-universally-necessary result which holds only under a condition on all four angle settings such as (11*a*), one must keep a watchful eye for the encroachment of surreptitious errors in interpretation. This required attention is further exacerbated in the situations we are addressing here, which involve the stipulation of the EPR locality premise. On account of the quantum mechanical stipulation of equations (8) and (9), the locality presumption must be carefully specified in the context of the four-particle electron experiment. For the system behavior of electron quartets explicitly *does* depend on the joint setting of all four angles. The magnetic spin behavior of an electron with respect to a magnet angle setting at station A explicitly does depend on the magnet angle settings at stations B, C, and D. This is the substance of equations (8) and (9). In this context, the premise of locality regarding experimental behavior must be understood to insist that no real change can take place in the behavior of any one of the electrons, at angle ϕ_1 say, in consequence of what is done in the other three, *presuming that the angles of the other three still ensure that all four angles yield the same value of the combination* $\phi_1 + \phi_2 - \phi_3 - \phi_4$. (My wording here closely modifies the wording of GHSZ in their statement of the Locality premise at the top right hand column of page 1134.)

It may seem that locality could be specified merely as relevant to an individual observation station angle, without reference to this qualification. However, if this specification were implied separately to all four angles without reference to the others, we would arrive in an impossible tangle of contradiction.

2.2.2 Thinking logically about contradictions

The problem is that the symmetry conditions yielding my equation (*iv*) pertain to experimental designs in which the linear combination of four magnet angles $\phi_1+\phi_2-\phi_3-\phi_4$ equals 0, whereas the angle combination proposed in equation (17) of GHSZ equals π, not equal to 0. In the first place, it is physically impossible to conduct a second experiment on the same quartet of electrons with the magnet orientations different from the conditions pertaining to the first. This cannot be done because those specific electrons have seen their day. It would be physically impossible even to isolate them together again. Of course we could *think about* what might have been the outcome of such a second experiment on the same electrons in a gedankenexperiment. That is the setting in which considerations of Bell's inequality are appropriate. However, even in such an *imagined* composite of experiments it would be a contradictory operation to set the angle combination to equal both 0 and to equal π in a single component experiment. Simultaneous application of (11*a*) and (11*b*) to the same component experiment would be contradictory.

Scientists and logicians are comfortable with the notion that presuming something which is logically impossible to be true can yield further contradictions, and also with the real awareness that it is impossible to perform a prescribed action whose operational description entails a contradiction. It is impossible to perform simultaneously a Stern-Gerlach quadruple electron experiments on a quartet of electrons with the angle settings satisfying two contradictory conditions. The restriction of the angle combination $\phi_1 + \phi_2 - \phi_3 - \phi_4$ to equal π is contradictory to the restriction that it equal 0. It is not surprising then that presuming them both will yield a contradiction.

Well, what then is the status of GHSZ equation (11b)?

2.3 The status of equation (11b)

Suppose we consider some implications of (11*b*) in a way comparable to the way GHSZ assess implications of (11*a*), line by line, even numbering our lines here identically to the GHSZ numberings and even using their wording. Remember that statement (11*b*) includes the premise that "If $\phi_1 + \phi_2 - \phi_3 - \phi_4 \;=\; \pi$", an alternate condition to that of the premise of (11*a*). Four instances of (11*b*) are

$$\begin{aligned} A_\lambda(\pi) \quad B_\lambda(0)\; C_\lambda(0)\; D_\lambda(0) &= 1, && (12a)\\ A_\lambda(\pi+\phi)\; B_\lambda(0)\; C_\lambda(\phi)\; D_\lambda(0) &= 1, && (12b)\\ A_\lambda(\pi+\phi)\; B_\lambda(0)\; C_\lambda(0)\; D_\lambda(\phi) &= 1, && (12c)\\ A_\lambda(\pi+2\phi)\; B_\lambda(0)\; C_\lambda(\phi)\; D_\lambda(\phi) &= 1. && (12d) \end{aligned}$$

From equations (12*a*) and (12*b*) we obtain

$$A_\lambda(\pi)C_\lambda(0) \;=\; A_\lambda(\pi+\phi)C_\lambda(\phi), \qquad (13a)$$

and from equations (12a) and (12c) we obtain

$$A_\lambda(\pi)D_\lambda(0) \;=\; A_\lambda(\pi+\phi)D_\lambda(\phi). \qquad (13b)$$

A consequence of these is

$$C_\lambda(\phi)/D_\lambda(\phi) \;=\; C_\lambda(0)/D_\lambda(0), \qquad (14a)$$

or equivalently,

$$C_\lambda(\phi)D_\lambda(\phi) \;=\; C_\lambda(0)D_\lambda(0), \qquad (14b)$$

since both $C_\lambda(\cdot)$ and $D_\lambda(\cdot)$ each equal ± 1, and thus equal their multiplicative inverses.

We then obtain from (12d) and (14b) the result

$$A_\lambda(\pi+2\phi)B_\lambda(0)C_\lambda(0)D_\lambda(0) \;=\; 1,$$

which in combination with (12a) yields

$$A_\lambda(\pi+2\phi) \;=\; A_\lambda(\pi) \;=\; const \;\text{ for all }\; \phi\,.$$

Again, as the denomination of an arbitrary angle by ϕ or by 2ϕ is irrelevant, this is equivalent to writing

$$A_\lambda(\pi+\phi) \;=\; A_\lambda(\pi) \;=\; const \;\text{ for all }\; \phi\,. \qquad (16b)$$

This result can be seen as companion to result (16) of GHSZ which is pertinent to the premise of the alternative condition (11a). Intentionally provocatively, I label this as equation (16b). GHSZ propose no equation (16b). It defies the result they portray as equation (18) which they derived by presuming contradictory premises. As a pair, the pleasing results (16) and (16b) beg for further discussion.

Assessing an experiment pertinent to (16b)

Remember again our agreement to support the entire theoretical development of GHSZ through equation (16). As to experimental considerations, let us forget for the moment the experiments we have assessed heretofore, and begin again afresh. Suppose we now design to perform the Stern-Gerlach experiment of GHSZ with a new quartet of electrons, prepared in the same entangled way as was the previous quartet. However, we shall now design the magnet directions $(\hat{\mathbf{n}}_1, \hat{\mathbf{n}}_2, \hat{\mathbf{n}}_3, \hat{\mathbf{n}}_4)$ so to specify the angle settings $(\pi, 0, 0, 0)$, motivating the assessment of $E^\psi(\hat{\mathbf{n}}_1, \hat{\mathbf{n}}_2, \hat{\mathbf{n}}_3, \hat{\mathbf{n}}_4) \;=\; -cos(\pi) \;=\; +1$. Now since $cos(-\theta) = cos(\theta)$ for any angle θ, the realm restrictions on this experimental design are equivalent to those on the design $(0, 0, \pi, 0)$, and since the angle 0 is equivalent to the angle 2π, these restrictions are equivalent to those on the design $(2\pi, 0, \pi, 0)$ as well. Thus, this experimental design $(\pi, 0, 0, 0)$ is equivalent to an instantiation of GHSZ's equation (11b) as $A_\lambda(2\pi)B_\lambda(0)C_\lambda(\pi)D_\lambda(0) \;=\; 1$.

The purpose of these machinations becomes apparent when we examine again the realm matrix for the measurement vector $(A_\lambda(\pi), B_\lambda(0), C_\lambda(0), D_\lambda(0))$ which will be recognisable as the matrix $\mathbf{R}_{+1}$ that we have already seen in equation (ii):

$$\mathbf{R}\begin{pmatrix} A_\lambda(\pi) \\ B_\lambda(0) \\ C_\lambda(0) \\ D_\lambda(0) \end{pmatrix} \;=\; \begin{pmatrix} 1 & -1 & -1 & -1 & 1 & 1 & 1 & -1 \\ 1 & -1 & 1 & 1 & -1 & -1 & 1 & -1 \\ 1 & 1 & -1 & 1 & -1 & 1 & -1 & -1 \\ 1 & 1 & 1 & -1 & 1 & -1 & -1 & -1 \end{pmatrix} \;\equiv\; \mathbf{R}_{+1} \quad .$$

The observed outcome of our experiment must equal one of the columns of this matrix. Finally, suppose that we consider still another design which would adjust the angle $\phi_1 = \pi$ at station A of the design above to an angle $\phi_1 = \pi + \theta$ for some arbitrary angle θ, while adjusting the angle ϕ_3 at station C from 0 to θ. The proposition of locality would ensure that the results of the spin measurements $B_\lambda(0)$ and $D_\lambda(0)$ would not be affected by this adjustment of the angles ϕ_1 and ϕ_3 in their distant localities. Concomitantly, equation (16b) assures that the value of $A(\pi+\theta)$ would remain equal to $A(\pi)$, *and* $C(\theta)$ would remain equal to $C(0)$, the results of the initial experiment.

Thus, EPR locality requires that whatever the observed results $B_\lambda(0), C_\lambda(0)$, and $D_\lambda(0)$ might be in our initial experiment, exhibited in one of the columns of $\mathbf{R}_+$, the value of $A_\lambda(\pi+\phi)$ would necessarily equal $A_\lambda(\pi)$ if the angle π at station A were switched to $\pi+\theta$ in the adjusted experiment *on the same quartet of electrons*! Of course making such a switch would necessitate switching the angle ϕ_3 at station C to θ to preserve the satifaction of (11b). The conclusion of (16b) would force the value of $C(\theta)$ in the adjusted experiment to remain equal to $C(0)$ in the initial one as well. This requirement derives from the fact that the product of any four column elements of the two relevant realm matrices must equal 1. Not only are the realm matrices identical for the original experiment and the adjusted one, but the actual result of the experiment on a specific electron quartet must be identical to that of the adjusted experiment. That is a consequence of locality, ensuring the spherical symmetry of the result of experiments satisfying (11b) on the same quartet of electrons.

Similar to the contradiction we discovered in Section 2.2 when we tried to presume the contradictory conditions of both $\phi_1 + \phi_2 - \phi_3 - \phi_4 = 0$ and $\phi_1 + \phi_2 - \phi_3 - \phi_4 = \pi$ on our experimental apparatus, we would achieve a matching contradiction if we now tried to presume both of these conditions here, and investigate their implications in the manner of GHSZ.

3 Technical conclusion

Quite to the contrary of the GHSZ conclusion that the premises of EPR pose a contradiction to a quantum experiment involving four (and even only three) particles, we can conclude that such experiments indeed do allow the premises of EPR. Furthermore, the conditions of such an experiment exemplify well the EPR premise of "Perfect correlation", in both the case of conditions of (11a) and that of (11b). ... though not at the same time! Smile.

4 Concluding comments

The results of this discussion have layed bare the claims of GHSZ who denigrate the logical consistency the EPR version of "local realism". However, I would not like a reader to think that I am in any way an advocate of the propositions that premise entails. I find it embedded in a view of physical experience that is seriously out of date. My investigations of the past two and a half years have been oriented, quite narrowly, to a resolution of the conundrum posed by the purported violation of Bell's inequality in the theory of quantum mechanics, in any of the forms in which it has been promoted. At a deeper level, I conclude that the so-called mysterious properties of the quantum world, involving a structure of "quantum probabilities" which inhere different structures than the mundane probabilities of the world at the classical scale, have been misconstrued. A discussion of larger implications relevant to a reconstruction of physical theory awaits a confirmation of my narrow concerns.

As to the definitive empirical work of Hensen et al. (2016), I do not doubt their experimental results. However, their statistical analysis suffers from the same mistake as does that of Aspect. Readers still impressed by it might review again Section 6 of Lad (2016).

I am indebted to several unnamed physicists who have graciously engaged with me in intense and detailed discussion of any number of technical issues about which we have disagreed. No one has heretofore seen the analysis which I have presented here. I have been challenged several times over the past year to address the GHSZ construct by colleagues who have been puzzled by my assessment of Aspect/Bell.

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