Impact of vortex core structure on equftion of motion

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Abstract

A problem allowing finding distribution of pressure (stress) in a quasi-solid hollow and continuous vortex core wall has been examined in this paper. Two other exact solutions derived from the linear theory strength of material are based on the idea of the task solutions: Lame's theory of thick-walled cylinders and task about the tube rotation. Comparison of the results obtained has been provided with known quantities. Some applications of the equations domains have been indicated.

Keywords: non-viscous liquid, distribution of stress, quasi-solid vortex core.

1. Introduction

The current task is considered within the framework of two models of fluid medium which came to be known as ideal and non-viscous liquid. Distinction between these models consists in the different amount of assumptions combined with Navier classic equation. In a model of the non-viscous medium it is considered that tangent tensions (friction) are absent only, and in the model of ideal liquid it is considered additionally that normal tensions in all directions are identical [1,2].

Among the large variety of flows within the framework of these models there is a class of the revolved streams containing two areas with the different laws of speed velocity along a radius. The first area being near-by the axis of rotation is called quasi-solid core as its angulator is permanent. The second area being between a core and immobile medium is characterized by large influence of viscid friction and moves with circuitous speed velocity according to a hyperbolic law. Any variant of such a flow is steady if the pressure near-by the axis of rotation is less than on periphery.

2. Reference data analysis

Classical description of the revolved stream is carried out on the basis of theory of potential flows and by means of dynamics of ideal liquid. As a result of the findings, it was defined that in the revolved stream the gradient of pressure in radial direction was counterbalanced by centrifugal force, i.e.

$$\frac{dp}{dr} = \rho\omega^2 r \, . \tag{1}$$

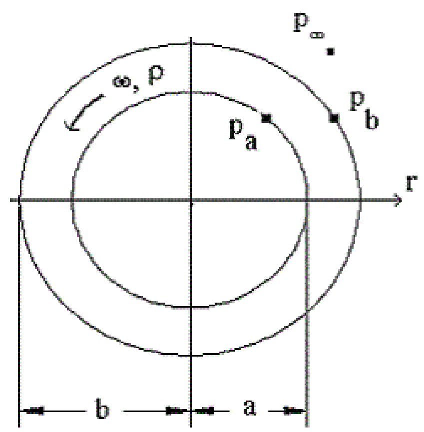
The variants of the use of this equation are considered in numerous works of theoretical and experimental character. A good compliance with the theory occurs at the rotation of stream into a tube or along with a tube, however such stream disintegrates quickly at expiration in an unlimited medium. The fact of disintegration of flow indicates at a huge impact of reaction of tube wall, however, it is not taken into account in equations [2, 3].

There are free revolved streams keeping up stability without a wall in radial direction, whose observation and the physical design find out the existence of hollow quasi-solid vortex core. Methods of mathematical description of dynamics of such core are semi empiric and do not have obvious connection with classic equations.

Thus, observation and experiment show that there are two structures of cylindrical core - continuous and hollow. In this scientific paper, the stress state of core of any structure within the framework of exact two-dimensional stationary task has been dealt with.

3. Statement of the problem, equations and analysis

In the given paper, quasi-solid area of rotation is considered a vessel which pressure is less than it is outside. In this case, in order to mathematically describe distribution of stress in a solid wall it is possible to use the equation known from the theory of resiliency for any structure of cross-sectional cylindrical wall [6].



Figs 1. Chart of denotations for the revolved ring

In denotations of theory of elasticity, the equation looks like:

$$\frac{d\sigma}{dr} + \frac{\sigma - \sigma_\theta}{r} + \rho\omega^2 r = 0, \quad (2)$$

where σ and σ_θ are radial and circuitous stress, respectively, p is core wall density , ω is an angular velocity.

Equation (2) has the following two particular solutions:

1. For an immobile cylindrical vessel while at pressures inside- (p_a) and outside (p_b) / *Lame's theory of thick-walled cylinders* /.

$$\sigma_r = \frac{p_a a^2 - p_b b^2}{b^2 - a^2} - \frac{p_b - p_a}{b^2 - a^2}\frac{a^2 b^2}{r^2},$$

$$\sigma_\theta = \frac{p_a a^2 - p_b b^2}{b^2 - a^2} + \frac{p_b - p_a}{b^2 - a^2}\frac{a^2 b^2}{r^2}$$

2. For the revolved tube:

$$\sigma_r = \frac{\rho\omega^2}{8}(3+\mu)\left(a^2 + b^2 - \frac{a^2b^2}{r^2} - r^2\right) \tag{3}$$

$$\sigma_\theta = \frac{\rho\omega^2}{8}(3+\mu)\left(a^2 + b^2 + \frac{a^2b^2}{r^2} - \frac{1+3\mu}{3+\mu}r^2\right), \tag{4}$$

where μ is Poisson's ratio taking into account properties of material and is determined by static tests results.

Therefore, in accordance with the laws of hydrostatics, tension does not depend on the orientation of surface element; a coefficient μ must be accepted as equal to unit.
Thus, for the quasi-solid vortex core, equation (3) and (4) will be like:

$$\sigma_r = \frac{\rho\omega^2}{2}\left(a^2 + b^2 - \frac{a^2b^2}{r^2} - r^2\right) \tag{5}$$

$$\sigma_\theta = \frac{\rho\omega^2}{2}\left(a^2 + b^2 + \frac{a^2b^2}{r^2} - r^2\right). \tag{6}$$

Let us define distribution of stress in the wall of rigid core which takes into account joint impact of pressures and rotation difference. As a task is dealt with within the framework of linear theory of resiliency, we will apply a theorem of superposition of solutions and we will get:

$$\sigma_r(p,\omega) = \frac{p_a a^2 - p_b b^2}{b^2 - a^2} - \frac{p_b - p_a}{b^2 - a^2}\frac{a^2b^2}{r^2} + \frac{\rho\omega^2}{2}\left(a^2 + b^2 - \frac{a^2b^2}{r^2} - r^2\right) \tag{7}$$

$$\sigma_\theta(p,\omega) = \frac{p_a a^2 - p_b b^2}{b^2 - a^2} + \frac{p_b - p_a}{b^2 - a^2}\frac{a^2b^2}{r^2} + \frac{\rho\omega^2}{2}\left(a^2 + b^2 + \frac{a^2b^2}{r^2} - r^2\right) \tag{8}$$

Equations (7) and (8) are correct for the quasi-solid core of any structure.
We will find the special case of distribution of stress in a continuous core. Then from (7) and (8) at $a = 0$.

$$\sigma_r(p,\omega) = \sigma_\theta(p,\omega) = -p_b + \frac{\rho\omega^2}{2}\left(b^2 - r^2\right). \tag{9}$$

At $r = b$, stress on the external wall of core becomes equal to pressure with a reverse sign.

One of differences of the revolved core from the rotation of solid tube is a presence of Bernoulli effect, as a result of pressure on an external surface of the core p_b is a less than pressure in medium p_∞ and it can be found from a formula $p_b = p_\infty - \frac{\rho\omega^2 b^2}{2}$ [1, 2].

By substituting this equation in (9) and taking into account the fact that tension and pressure have reverse signs, we will get:

$$p(r) = p_\infty - \frac{\rho\omega^2}{2}\left(2b^2 - r^2\right), \qquad (10)$$

where $r = 0...b$.

Equations (9) and (10) coincide with the one obtained in the dynamics of ideal liquid.

In a fig. 2 shown the example of distribution of radial and circuitous tensions for a core $b = 5a$, built in a Mathcad package.

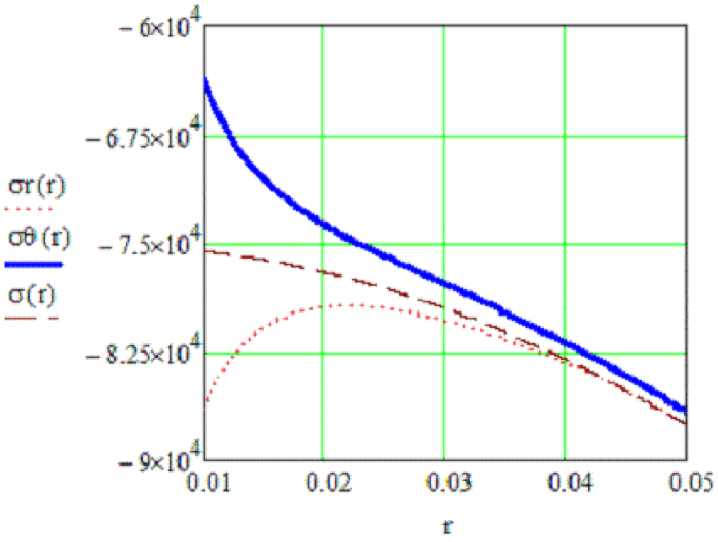


Fig. 2. Distribution of radial and circuitous tensions in a rigid core (density of $\rho = 1000$ *kg/ м*[3], angular velocity $\omega = 100$ *1/c*, and $a = 0.01$ *m*,
b = 0.05 m, $p_\infty = 10^5$ *Pa*). A chart in the middle is built on equation (9).

It follows from a chart that circuitous stress exceed radial and with the increase in relative thickness of vortex, both tensions tend to one value.

Equation (8) by the data $\sigma_\theta \geq 0$ allows finding rotations parameters at which circuitous tension on an internal surface will become stretching and the flow will begin to disintegrate. These data can be used to fight against craters that are available or to prevent their origin. Then at $r = a$ wewill get that the angular velocity of internal surface of core can be calculated by equation.

$$\omega_a = \sqrt{\frac{2p_b - p_a\left(1 + \frac{a^2}{b^2}\right)}{\rho b^2\left(1 - \frac{a^2}{b^2}\right)}}$$

If angular velocity of the core ω will exceed the value *of* ω_a, circuitous tensions will become stretching.

Equation (8) by the data $\sigma_\theta < 0$ can be used to estimate the stability of core existence at the possible modes of rotation. Such a task arises up while using positive properties of flow and its application, for example, for a separation or jointly with the processes of heat exchange. It is of importance to use viscid secondary flow which emerges while braking butt end of the vortex core on any surface [3, 7].

4. Conclusions

4.1 The task solution about the distribution of stress in the quasi-solid wall of cylindrical vortex core uses two known exact solutions of Navier equation.
4.2 Equation (2) can be used for the core of any structure, and equation (1) is intended only for a continuous vortex.
4.3 The special case of the found equations coincides with the known distribution of pressures for the vortex of ideal liquid.
4.4 The above equations do not take into account the influence of viscid friction that has substantial impact upon the area of flow near-by wall.

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5. References

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