Dynamical Construction of Glueballs in Pure Gluodynamics

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Abstract

The correlation of the quantization problems in both canonical approach and path-integral method in the pure Yang – Mills fields for strong interactions leads to the conclusion that Gribov copies and virtual gluons in QCD vacuum are one and the same thing. This conclusion leads to dynamical construction of glueballs that explains color confinement and mass gap problems in pure gluodynamics.

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1. INTRODUCTION:

We analyze the canonical structure of pure Yang-Mills theory in the case when we satisfy Gauss' law identically by construction. Here the boundary conditions play critical role in the emergence of a canonical structure. It is shown that the theory has multiple short-lived canonical structures for same physical situation in this case, provided one uses a generalized Coulomb gauge condition for infinitesimal time period and takes into account Gribov ambiguity. The same Gribov ambiguity is shown to yield to multiple representation of any physical gauge orbit in path-integral quantization also. Given the fact that both canonical and path-integral approaches must yield equivalent quantization results [1, 2] for QCD, it is but natural to expect some sort of correlation between the aforesaid quantization problems of these approaches and the same is derived in this paper.

2. CANONICAL APPROACH

For the initial formulation of the classical Yang – Mills equations, I would refer to [3] in this and subsequent para. Classically, the gauge theory is illustrated by the theory of electromagnetism for which the aeblian group $U(1)$ is the gauge group. If we locally view the $U(1)$ gauge connection, as denoted by A, as one-form on space-time, then the two form $F = dA$ is nothing but curvature or electro-magnetic field tensor in space-time. In this terminology, the Maxwell's equations are represented by $0 = dF = d{*F}$, where * is the Hodge duality operator [3].

Classically, by substituting the abelian group $U(1)$ with a more general compact gauge group $G = SU(3)$, the curvature of the space-time is changed to $F = dA+A\wedge A$, and Maxwell's equations are transformed to the Yang-Mills equations, $0 = d_A F = d_A{*F}$, where d_A denotes the gauge-covariant extension of the exterior derivative. These Yang-Mills equations can be validated by deriving them from the pure Yang-Mills Lagrangian [3]

$$L' = (1/4g^2) \int \mathrm{T_r}\, F\wedge{*F} \qquad (1)$$

where $\mathrm{T_r}$ denotes an invariant quadratic form on the Lie algebra of G.

In the relativistic Lagrangian of Equation (1), the time and space indices have been treated on equal footing. If the time index, as a parameter, is treated on unequal footing with space indices in the Lagrangian of Equation (1), we get new non-relativistic Lagrangian L of space indices as

$$L' = \int_{t_1}^{t_2} dt\, L \qquad (2)$$ where integration is from time instant t_1 to t_2.

The very first step towards canonical quantization involves the conversion of the classical Lagrangian L of Equation (2) into a Hamiltonian one. The standard procedure [4] for this is to define canonical momenta π^{μ} by the following mathematical relations.

$$\pi^{\mu} = \partial L/\partial \acute{A}_{\mu} \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_} \quad (3)$$

where $\mu = 0,1,2,3$ indicates the Minkowski space-time indices and $\acute{A}_{\mu}$ are the generalized velocities.

Now, the canonical momenta π^0 corresponding to the time index $\mu = 0$ in the above mathematical formula vanishes in [4] at the outset of the quantization process itself in the Dirac formulation for the constrained systems [5, 6]. This fundamental mathematical problem of the vanishing canonical momenta π^0 originates from the mere fact that for the non-relativistic Lagrangian L of Equation (2), the time index $\mu = 0$ acts as a parameter in the time interval (t_1, t_2) while the space indices ($\mu = 1, 2, 3$) are acting as independent variables. With $\pi^0 = 0$, the time-dependent gauge transformation of A_0 can be fixed only in the dynamics of the remaining gauge field variables.

Let us see how this happens: Neglecting the problem of the vanishing canonical momenta π^0, in the light of the time index being chosen as a parameter to begin with, if the corresponding pairs of spatial components of Yang – Mills gauge field A_k (where the space indices k = 1,2,3) and canonical momentas π_k (where the space indices k = 1,2,3), corresponding to spatial co-ordinates, are chosen as canonically conjugate variables in the non-relativistic Lagrangian L of the Equation (2), it is possible to mathematically construct [4] a Hamiltonian H through the Legendre transformation given as

$$H = \int \pi_k . \acute{A}_k \, d^3x - L \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_} \quad (4)$$

The above H, through the Hamiltonian equations of motion, reproduces [4] the dynamical part of the Yang – Mills equations but Gauss's law is absent in such a system. No physical applications, like working out the physical particle spectrum of QCD from the Hamiltonian H, are possible before Gauss law is properly incorporated into the Hamiltonian formalism [4].

It is shown in [7] that the starting point, for the implementation of the Gauss law in Hamiltonian formalism, is the non-abelian Gauss law that is obtained as a Lagrange equation of motion for $\mu = 0$ when Hamilton's action principle is invoked for the relativistic Lagrangian of Equation (1), i.e.,

$$\nabla_k(A)\nabla^k(A)A^0 - \nabla_k(A)\,\acute{A}^k = 0 \qquad (5)$$

In-fact, with time index $\mu = 0$ acting as a parameter in the time interval (t_1, t_2) and with space indices ($k = 1, 2, 3$) acting as independent variables, it is mentioned in [7] that the aforementioned non-abelian Gauss law in Equation (5) is a system of linear, elliptic partial differential equations determining the (matrix valued) potential component A_0 for given space components A_k & their time derivatives $\partial_0 A_k$ and it can be solved by assuming the existence of unique solution A_0 as a functional of A_k and their time derivatives $\partial_0 A_k$, i.e.,

$$A_0 = A_0\{A_k, \partial_0 A_k\} \qquad (6)$$

At this stage, a natural question, as raised in [7], is then whether the pure Yang – Mills Lagrangian L' of the Equation (1) can be used for deriving a canonical structure of the pure Yang – Mills when the potential component A_0, as given by the Equation (6) above, is a solution of the aforementioned non-abelian Gauss law in Equation (5). For answering this question, it is mentioned in [7] that the first step is to use the Lagrangian L of the

Equation (2) that has been derived from *L'* to begin with and then substitute the aforesaid unique solution A_0, as given by the Equation (6) above, into this L to get new Lagrangian L_0 as given below,

$$L_0 = (-1/2) \int_V d^3x\, (\nabla_k(A_k)\, A_0\{A_k, \partial_0 A_k\} - \acute{A}_k, \nabla^k(A_k)\, A_0\{A_k, \partial_0 A_k\} - \acute{A}^k)$$
$$- (1/4) \int_V d^3x (G_{kl}(A_k), G^{kl}(A_k)) \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_}(7)$$

where $G_{kl} = \partial_k A_l(x) - \partial_l A_k(x) - ig[A_k(x), A_l(x)]$ and k,l are denoting space indices ranging from 1 to 3.

This new Lagrangian L_0 must reproduce the Lagrange equations of motion for k = 1,2,3 when Hamilton's action principle is invoked. Towards this goal, it is quite obvious to verify the following result beforehand as is done in [7],

$$\delta \int dt\, L_0 = - \int dt \int_V d^3x\, (\delta A_k, \nabla_0(A)\, (\nabla^k(A)\, A^0\{ A_k, \partial_0 A_k\} - \acute{A}^k) - \nabla_l G^{kl}(A))$$
$$- \int dt \int_{\partial V} d^2\sigma_k\, ((\delta A^0\{ A_k, \partial_0 A_k\}, \nabla^k(A)\, A^0\{ A_k, \partial_0 A_k\} - \acute{A}^k) \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_} (8)$$

where time integration is from time instant t_1 to t_2.

Now, it is mentioned in [7] that the boundary conditions applicable to the A_0 are required to be considered at this stage. If the domain V is taken as all R^3 and accordingly, the boundary of the domain V in above Equation (8) is taken at spatial infinity ($R \to \infty$), then the vanishing of the second surface term in above Equation (8) is taken to be equivalent to the following condition in [7],

$$\lim_{R\to\infty} \int_{V,\ |x| = R} d\Omega R^2\, (\delta A^0\{ A_{(r)}, \partial_0 A_{(r)}\}, \nabla^{(r)}(A)\, A^0\{ A_{(r)}, \partial_0 A_r\} - \acute{A}^{(r)}) = 0 \quad \text{\_\_\_\_\_\_\_\_} (9)$$

where (r) denotes the radial component of the corresponding quantity.

As such, subject to the above boundary condition in Equation (9) for all admissible variations of $A_{(r)}$ and $\partial_0 A_{(r)}$, we get the following Lagrange equations of motion for k = 1,2,3 for L_0 from Equation (8) in the light of the variational principle $\delta \int dt\, L_0 = 0$, i.e.,

$$\nabla_0(A)\,(\nabla^k(A)\,A^0\{A_k, \partial_0 A_k\} - \acute{A}^k) - \nabla_l G^{kl}(A)) = 0 \qquad (10)$$

Further, it is mentioned in [7] that the vanishing of the surface term in the Equation (9) above depends upon the assumed asymptotic behavior of the independent variables A_k and their time derivatives $\partial_0 A_k$ as well as the on the boundary conditions (at spatial infinity) of the dependent variable A_0.

As such, given that the surface term in the Equation (9) above does vanish under these asymptotic and boundary conditions being fulfilled and accordingly, the L_0 does reproduce the Lagrange Equations (10) of motion for k = 1,2,3, then the substitution of L_0 of the Equation (7) into the Equation (4) above leads to the generalized velocity-dependence of the Hamiltonian H through the dependence of A_0 on the time derivatives $\partial_0 A_k$ in L_0. But we must eliminate somehow the aforesaid generalized velocity-dependence of Hamiltonian H, or in other words the dependence of A_0 on the time derivatives $\partial_0 A_k$, if we intend to have canonical structure of the pure Yang – Mills theory at all.

This elimination can be accomplished by imposing, in Equation (5), an attainable gauge fixing i.e., for every Yang – Mills field configuration, there must exist a gauge-transformed Yang – Mills field configuration that satisfies the following generalized coulomb gauge fixing condition [7], i.e.,

$$\nabla_k(A)\,\acute{A}^k = 0 \qquad (11)$$

However, it is mentioned in [7] that with the use of the above generalized coulomb gauge fixing condition of the Equation (11), the generalized velocities $\partial_0 A_k$ are no longer independent quantities and as such, cannot be used for the construction of canonical momentas in the Equation (3). As such, an alternate line of reasoning that leads to a proper canonical formalism is given below.

If the aforesaid generalized coulomb gauge fixing is assumed to come into play only for an **infinitesimal time period**, then the generalized velocities $\partial_0 A_k$ can be treated as independent quantities for all intent & purpose and can be used for the construction of canonical momentas in the Equation (3). Also, the implementation of the non-abelian Gauss law in Hamiltonian formalism is only for this infinitesimal time period during which the aforesaid elimination of the dependence of A_0 on the time derivatives $\partial_0 A_k$ by the generalized coulomb gauge fixing condition persists. Accordingly, the Hamiltonian H of the Equation (4) exists in time-independent form only for this infinitesimal time period because A_0, the one and only one temporal component contained in the Hamiltonian H, can be assumed to stay constant during this infinitesimal time period.

This implies that the physical applications, like working out the physical particle spectrum of QCD from this short-lived Hamiltonian H, can only yield short-lived massless gauge bosons. Further, although the dynamical time-dependent gauge freedom of the Hamiltonian H can be fixed only for infinitesimal time period during which A_0, the temporal component of the Yang – Mills gauge field for the strong interactions, is assumed to stay constant, but there still remains during this infinitesimal time-period the freedom of performing time-independent gauge transformations in the Hamiltonian H that has been constructed, at first instance, from spatially dependent canonically conjugate variable pairs.

Apparently, these time-independent gauge transformations shatter the uniqueness of the Hamiltonian H as it is not possible to fix these time-independent gauge transformations in the light of Gribov ambiguity. Thus, it is obvious that when transition to quantum version of the Hamiltonian H by means of fixed time Schrödinger quantization rule is done, two or more short-lived massless gauge bosons exists in one to one correspondence with the Gribov copies for any physical application of H.

3. PATH INTEGRAL METHOD

Further, we would now take the case of path integral method for quantization of classical Yang – Mills theory for strong interactions. In this path-integral method, the vacuum-to-vacuum transition amplitude is given by,

$Z = \int [dA] \exp. [i \int d^4x\, L(A)]$ where $\int d^4x\, L(A) = L'$ and $i = (-1)^{1/2}$ __________________ (12)

"The integral space of the gauge fields in the integral $\int dA$ in the equation (12) above can be visualized as a product of integral length of a full set of gauge-inequivalent (i.e., gauge-fixed) configurations in the integral $\int dA^{g.f.}$ and integral length over the gauge group $\int d_g$. In other words, the integral length in the integral $dA^{g.f.}$ corresponds to the set of all possible gauge orbits and the same in the integral $\int d_g$ refers to the length of the gauge orbits. As such, we can conclude

$$\int dA \equiv \int dA^{g.f.} \int d_g$$

To make integrals such as those in the Eq. (12) finite and also to study gauge-dependent quantities in a meaningful way, we need to eliminate this integral around the gauge orbit, $\int d_g$.

The Faddeev-Popov gauge-fixing procedure eliminates this integral around the gauge orbit, $\int d_g$ in the perturbative QCD and in that way, it leads to ghosts and the local BRST invariance of the gauge-fixed perturbative QCD action. Since, for small field fluctuations of the perturbative QCD in the asymptotic regime, the Gribov copies [8] cannot be conscious of each other, so they can be neglected. But this situation does not the same in the non-perturbative QCD. Accordingly, the definition of the non-perturbative QCD should be such that the functional integral (12) contains each gauge orbit only once in order to eliminate the aforementioned integral around the gauge orbit, $\int d_g$. In other words, the non-perturbative QCD is to be defined in such a way that it has no Gribov copies. An implicit assumption in lattice QCD studies is to define the non-perturbative QCD in this way.

The generalized Faddeev-Popov technique is used for arriving at this definition of the nonpertubative QCD and in that way, just one gauge configuration on the gauge orbit is not chosen but rather what actually chosen is some Gaussian weighted average over the gauge fields on the gauge orbit. In the light of this choice, a non-local action $\int d^4x\ L(A)$ and a non-local quantum field theory arises in this definition of the nonpertubative QCD. But the proof of the renormalizability of QCD, the proof of asymptotic freedom, local BRST symmetry, and the Schwinger-Dyson equations etc. are obtained on the basis of this action $\int d^4x\ L(A)$. So, when this action $\int d^4x\ L(A)$ becomes non-local, these features of QCD cannot be proved in the non-perturbative context due to the absence of reliable basis.

In other words, the basic features like locality and the BRST invariance of the QCD theory stand shattered in this definition of the nonpertubative QCD. But many authors [9 -12] in the literature have upheld an equally valid viewpoint that QCD must be defined with locality and BRST symmetry at the centre and these features should not be given up while defining QCD in the nonpertubative sector. The implications of this viewpoint are that the presence of Gribov copies is absolutely necessary, that the multiple representations of the Gribov orbits are a fact."[13]

In-fact, the aforementioned indispensable presence of the Gribov copies in the definition of nonperturbative QCD forces one to search for the physical interpretation of these Gribov copies. As depicted in the Figure.1 of [13] above, the "gauge orbit" for some configuration A is defined to be set of all gauge-equivalent configurations, and by definition, the action $\int d^4x\ L(A)$ is gauge invariant, so all the configurations A' on the gauge orbit have got same action.

This implies that the Gribov copies, in each gauge orbit, correspond to one and same physical situation such that there is residual gauge freedom given by gauge transformation between these Gribov copies. For exhibiting this residual gauge freedom in the light of the aforementioned indispensable presence of the Gribov copies in the definition of nonperturbative QCD, the necessary and sufficient condition is that the representative gauge potential on each of the gauge orbit in the physical configuration space (the space of all gauge orbits) must be surrounded by all the Gribov copies of that gauge orbit in Minkowski space-time. In other words, any real gluon as a representative gauge potential on each of the gauge orbit in the physical configuration space must be surrounded by massless gauge bosons, each referring to one of the Gribov copies of that gauge orbit in Minkowski space-time.

4. CORRELATION

As pointed out earlier, the quantization results of the canonical approach and the path-integral method must be equivalent. In the light of the forgoing statement, we can correlate the quantization results of these two methods by asserting that the massless gauge bosons, surrounding the real gluon and each referring to one of the Gribov copies of a gauge orbit in Minkowski space-time are **short-lived** one.

5. CONCLUSION

Due to vacuum polarization, a real gluon is surrounded by massless virtual gluons that are also short-lived one in the Minkowski space-time. This leads to contradiction that the ground state or the vacuum state of the classical Yang – Mills field seemingly consists of two sets of massless particles: one corresponding to the Gribov copies and the other massless virtual gluons. The one and only one way to resolve this contradiction is to conclude that the Gribov copies and massless virtual gluons in QCD vacuum are one and the same thing.

6. DISCUSSION:

The above conclusion implies that a real gluon, created as one of three jets emerging from electron-positron annihilation at high energy at L3 Collaboration, CERN [14] at any point of time, would be immediately surrounded by its Gribov copies in the QCD vacuum.

In other words, this real gluon and its Gribov copies in QCD vacuum are multiply representing a particular gauge orbit that exists in the space of gauge potentials. If we envisage the gauge field of this particular gauge orbit, that is multiply represented by the real gluon and its Gribov copies, being used to perturbatively dress the matter field of a perturbative static Lagrangian quark, we can easily construct gauge-invariant colored static quark in perturbation theory for the mere fact [13] that here standard gauge fixing is unique for a gauge orbit as Gribov copies cannot be aware of each other for small field fluctuations and can be easily neglected for large momenta. Indeed, classical QCD is scale-invariant as there are no natural scales in it.

But a confining scale automatically gets generated in non-perturbative QCD at such a spatial locality when the Gribov copies, representing the same gauge orbit, become aware of each other and start interchanging their representative role by exhibiting residual gauge transformation amongst them. At this spatial locality, we can cope with the insensitivity [15] of the dressing (of gauge orbit) towards matter field color (of static Lagrangian quark) at the place of occurrence of the Gribov copies on this particular gauge orbit by postulating different color charges to the Gribov copies so that the overall color charge of the matter field and dressing is preserved on the whole of this particular gauge orbit.

Since, there is no inherent preference for any specific color charge so far as the representation of that particular gauge orbit by real gluon and its Gribov copies in QCD vacuum is concerned, so, it is perfectly straightforward to consider the ensemble of the real gluon and its Gribov copies in QCD vacuum as color-singlet one. Further, this color-singlet ensemble of the real gluon and its Gribov copies is physically co-existent in QCD vacuum as multiple representatives of the same gauge orbit, so, the static matter field, that is being dressed with the gauge field of this gauge orbit, also needs to be color-singlet one. This explains why the matter field of colored quark is forbidden to be dressed with the gauge field of the gauge orbit in non-perturbative QCD and the color-singlet hadron is allowed as physical observables after combined matter field of its constituents is dressed as a whole with the gauge field of the gauge orbit in non-perturbative QCD. This is nothing but confinement mechanism of quarks in non-perturbative QCD.

Next, the question arises how mass gap is generated in the above scenario. In pure gluodynamics, the above explained color-singlet ensemble of the real gluon and its Gribov copies is physically co-existent in QCD vacuum as multiple representatives of the same gauge orbit. At perturbative level, the Gribov copies, as already stated, are not aware of each other and consequently, all the real gluons remain massless. However, this normal massless vacuum is unstable, since each and every real gluon and their corresponding Gribov copies become aware of each other and start interacting with each other to form color-singlet ensemble called Gribov glue ball at such a spatial locality when confining scale automatically gets generated at non-perturbative level.

Consequently, stable vacuum is realized after the condensation of these Gribov glue balls. The excitation spectrum has a dynamical mass gap generation because the constituent real gluon and its Gribov copies in any color-singlet Gribov glue ball continuously exhibit residual gauge transformation amongst themselves by interchanging their respective roles as multiple representatives of the common gauge orbit. The amount of dynamical mass gap generated in any Gribov glue ball depends upon the largeness of gauge field that is carried by its respective gauge orbit.

Lastly, the mechanism for phase transition from confinement to deconfinement with increasing temperature needs to be probed in above scenario of Gribov glue balls. In pure gluodynamics, the confined phase and deconfined phase are distinguished by an order parameter called the expectation value of the Polyakov loop [16] that is zero for the confined phase and non-zero for the deconfined one. Naturally, Gribov glue balls exist in confined phase. From symmetry point of view, the Polyakov loop characterizes the breaking of Gribov glue balls. At critical temperature, the real gluon and its Gribov copies starts losing contact with each other for small field fluctuations and this ultimately causes disassociation of Gribov glue balls above critical temperature when phase transition from confined phase (of Gribov glue balls) to deconfined phase (of gluon plasma) takes place.

REFERENCES:

[1] G. C. Rossi, M. Testa, Nucl. phys. B163 (1980) 109

[2] H. Reinhardt, Mod. Phys. Lett. A11 (1996) 2451

[3] A. Jaffe and E. Witten, Quantum Yang-Mills Theory (Clay Mathematics Institute Millennium Problem, 2000).

[4] A. Salmela: Gauss's law in Yang – Mills theory, University of Helsinki, 2005, 57 p, University of Helsinki, Report Series in Physics, HU-P-D126, ISSN 0356-0961, ISBN 952-10-2107-1 (printed version), 952-10-2108-X (PDF)

[5] P.A.M. Dirac, Canad. J. Math. 2, 129 (1950)

[6] P.A.M. Dirac, Lectures on Quantum Mechanics, Benjamin, New York, 1964

[7] C. Cronström, Canonical structure and Boundary Conditions in Yang-Mills Theory, Preprint HIP-1999-33/TH [eprint archive hep-th 99/9906184v1]

[8] V. N. Gribov, Nucl. Phys. B 139 (1978) 1.

[9] H. Neuberger, Phys. Lett. B 183, 337 (1987).[4]

[10] M. Testa, Phys. Lett. B 429, 349 (1998) [arXiv:hep-lat/9803025].

[11] M. Testa, arXiv:hep-lat/9912029.

[12] R. Alkofer and L. von Smekal, Phys. Rept. 353, 281 (2001) [arXiv:hep-ph/0007355] and references therein.

[13] A. G. Williams, arXiv:hep-lat/0202010v1.

[14] Frank Wilczek, Asymptotic Freedom: From Paradox to Paradigm, Lecture given in acceptance of Noble Prize, December, 2004

[15] A. Ilderton, M. Lavelle and D. McMullan, JHEP 0703 (2007) 044 [arXiv:hep-th/0701168].

[16] R. Casalbuoni, arXiv:hep-ph/0610179v1.