Critical phenomena of nuclear matter in the extended Zimanyi-Moszkowski model

K. Miyazaki*

Abstract

We have studied the thermodynamics of warm nuclear matter below the saturation density in the extended Zimanyi-Moszkowski model. The EOS behaves like van der Waals one and shows the liquid-gas phase transition as the other microscopic EOSs. It predicts the critical temperature $T_C = 16.36\,\mathrm{MeV}$ that agrees well with its empirical value. We have further calculated the phase coexistence curve and obtained the critical exponents $\beta = 0.34$ and $\gamma = 1.22$, which also agree with their universal values and empirical values derived in the recent experimental efforts.

The equations of state (EOSs) of warm nuclear matter derived by microscopic theories generally exhibit the similar nature to the van der Waals EOS. This indicates that the liquid-gas phase transition occurs in warm nuclear matter or that no nucleus as a droplet surrounded by vapor exists above the critical temperature T_C. In fact the phase transition has been observed in nuclear multifragmentation reactions and the critical temperature has been derived as $T_C = 20 \pm 3\,\mathrm{MeV}$ in Ref. [1] and $T_C \approx 8\,\mathrm{MeV}$ in Ref. [2]. Unfortunately, the results strongly depend on the models used in the analyses. However we have to note [3] that the critical point does not possess a direct correspondence in heavy finite nuclei because of the Coulomb interaction and the finite-size effect. The maximal temperature at which a nucleus can be observed in experiments is the limiting temperature but not the calculated critical one itself. Nevertheless, the ratio between the two temperatures is fairly stable and independent on the particular EOS and the method used in analysis. This allowed Natowitz *et al.* [4] to estimate the critical temperature for infinite nuclear matter, that is, $T_C = 16.6 \pm 0.86\,\mathrm{MeV}$. It is this value to be compared with various models of nuclear matter. In our opinion, at present, no theories however can reproduce the value successfully.

In the present work we apply the new nonlinear relativistic mean-field model [5,6] to warm symmetric nuclear matter. The model is an extension of the Zimanyi-Moszkowski model [7] based on the constituent quark picture of nucleon. The nucleon structure is reflected in the renormalized meson-nucleon coupling constants that depend on the effective mass of a nucleon in the medium. Our model, hereafter called as the extended Zimanyi-Moszkowski (EZM) model, can reproduce the nuclear mater saturation properties that are comparable to the Dirac-Brueckner-Hartree-Fock (DBHF) theory [8]. We

*E-mail : miyazakiro@rio.odn.ne.jp

can therefore believe that the EZM model is reasonable at zero temperature. However this never means that the model is useful at finite temperature. In fact the DBHF calculation of warm nuclear matter [9] predicts the critical temperature $T_C = 10.4\,\mathrm{MeV}$ that is rather lower than the empirical value. It is therefore necessary to test the applicability of the EZM model to the liquid-gas phase transition of nuclear matter.

The thermodynamic potential per volume $\tilde{\Omega} \equiv \Omega/V$ in the EZM model of symmetric nuclear matter at finite temperature T is

$$\begin{aligned}\tilde{\Omega} &= \frac{1}{2} m_\sigma^2 \langle\sigma\rangle^2 - \frac{1}{2} m_\omega^2 \langle\omega_0\rangle^2 \\ &- \gamma\, k_B T \int_0^\infty \frac{d^3\mathbf{k}}{(2\pi)^3} \left\{ \left[1+\exp\left(\frac{\nu - E_k^*}{k_B T}\right)\right] + \left[1+\exp\left(\frac{-\nu - E_k^*}{k_B T}\right)\right] \right\}, \end{aligned} \tag{1}$$

where k_B is the Boltzmann constant and $E_k^* = (\mathbf{k}^2 + M^{*2})^{1/2}$ with the effective mass M^* of a nucleon in the medium. For symmetric nuclear matter the spin-isospin degeneracy is $\gamma = 4$. The ν is defined by the chemical potential μ and the vector potential V_0 of a nucleon as

$$\nu = \mu - V_0. \tag{2}$$

The scalar mean field $\langle\sigma\rangle$ is determined from the effective mass by

$$\langle\sigma\rangle = \frac{M - M^*}{g^*_{NN\sigma}} = \frac{(1 - m^*)}{g^*_{NN\sigma}} M, \tag{3}$$

where M is the free nucleon mass and $M^* = m^* M$. The renormalized $NN\sigma$ coupling constant [6] is

$$g^*_{NN\sigma} = [(1-\lambda) + \lambda\, m^*]\, g_{NN\sigma} \tag{4}$$

with

$$\lambda = 1/3. \tag{5}$$

It is noted that $\lambda = 0$ corresponds to the Walecka model [10] while $\lambda = 1$ corresponds to the original ZM model [7]. The renormalized $NN\omega$ coupling constant [6] is also given by

$$g^*_{NN\omega} = [(1-\lambda) + \lambda\, m^*]\, g_{NN\omega}. \tag{6}$$

The vector mean field $\langle\omega_0\rangle$ is determined from the vector potential as

$$\langle\omega_0\rangle = \frac{V_0}{g^*_{NN\omega}} = \frac{v_0}{g^*_{NN\omega}} M. \tag{7}$$

Then, the effective mass m^* and the vector potential v_0 are determined from extremizing the thermodynamical potential $\tilde{\Omega}$ by them. We have

$$m^* = 1 - \left(\frac{g_{NN\sigma}}{m_\sigma}\right)^2 \left\{ [(1-\lambda) + \lambda\, m^*]^3 \, \frac{\rho_S}{M} + \lambda \left(\frac{m_\omega}{g_{NN\omega}}\right)^2 v_0^2 \right\}, \tag{8}$$

$$v_0 = \left(\frac{g^*_{NN\omega}}{m_\omega}\right)^2 \frac{\rho_B}{M}, \tag{9}$$

where the baryon and scalar densities are

$$\rho_B = \gamma \int_0^\infty \frac{d^3\mathbf{k}}{(2\pi)^3} \left[n_k\,(T) - \bar{n}_k\,(T) \right], \tag{10}$$

$$\rho_S = \gamma \int_0^\infty \frac{d^3\mathbf{k}}{(2\pi)^3} \frac{M^*}{E^*_k} \left[n_k\,(T) + \bar{n}_k\,(T) \right]. \tag{11}$$

The Fermi-Dirac distribution functions of nucleon and antinucleon are

$$n_k\,(T) = \left[1 + \exp\left(\frac{E^*_k - \nu}{k_B T} \right) \right]^{-1}, \tag{12}$$

$$\bar{n}_k\,(T) = \left[1 + \exp\left(\frac{E^*_k + \nu}{k_B T} \right) \right]^{-1}. \tag{13}$$

The energy density is given by

$$\mathcal{E} = \frac{1}{2}\left(\frac{m_\sigma}{g^*_{NN\sigma}}\right)^2 (M - M^*)^2 + \frac{1}{2}\left(\frac{g^*_{NN\omega}}{m_\omega}\right)^2 \rho_B^2 + \gamma \int_0^\infty \frac{d^3\mathbf{k}}{(2\pi)^3} E^*_k \left[n_k\,(T) + \bar{n}_k\,(T) \right], \tag{14}$$

and the pressure is

$$P = -\tilde{\Omega}. \tag{15}$$

Given the baryon density ρ_B and the temperature T, Eqs. (8), (9) and (10) have to be solved numerically so that the effective mass m^*, the vector potential v_0 and the chemical potential μ are determined self-consistently. Figure 1 shows the pressure-density isotherms of symmetric nuclear matter below the saturation density at several temperatures. It exhibits a typical van der Waals nature, that is, the liquid-gas phase transition. The temperature $T = 12.87\,\mathrm{MeV}$ is the flash temperature above which the pressure is always positive at any density. We have found the critical temperature $T_C = 16.36\,\mathrm{MeV}$, where the corresponding $P - \rho_B$ isotherm has an inflection point at $P_C = 0.308\,\mathrm{MeV{\cdot}fm^{-3}}$ and $\rho_C = 0.059\,\mathrm{fm^{-3}}$.

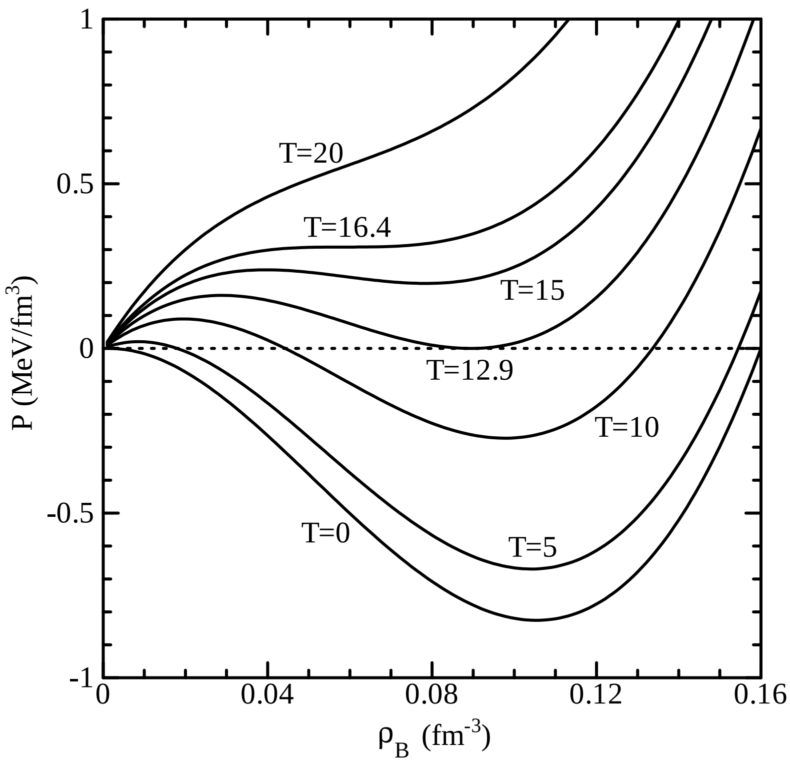

Figure 1: The pressure-density isotherms of symmetric nuclear matter below the saturation density at several temperatures.

The obtained critical temperature agrees well with the empirical value $T_C = 16.6\,\mathrm{MeV}$. In this respect, we will compare the EZM model with the other models. As already mentioned, although both the EZM and DBHF models are able to reproduce well the nuclear matter saturation properties with the effective mass $m^* = 0.6$ and the incompressibility $K \simeq 300\,\mathrm{MeV}$, the DBHF cannot reproduce the empirical value of T_C. On the other hand, the nonrelativistic Brueckner-Hartree-Fock (NRBHF) calculation [11] with and without including three-body force predicts $T_C \approx 16\,\mathrm{MeV}$ and $T_C \approx 13\,\mathrm{MeV}$ respectively. It is noted that the three-body force is necessary to reproduce the nuclear matter saturation in the NRBHF. We therefore conclude that the NRBHF cannot reproduce the empirical value of T_C.

Next, we review the relativistic mean-field models. The original Walecka model [10] predicts $T_C = 18.3\,\mathrm{MeV}$ that is somewhat larger than the empirical value, while its nonlinear extension [12] predicts $T_C = 14.4\,\mathrm{MeV}$. Both the models are not satisfactory. Although the original ZM model [13] predicts $T_C = 16.5\,\mathrm{MeV}$ that reproduces the empirical value fairly well, its effective mass $m^* = 0.85$ of a nucleon is too large to reproduce spin-orbit splitting of finite nuclei [14,15]. On the other hand, the other versions of the ZM model [16] cannot predict m^* nor T_C well. In this respect, the EZM model can be regarded as a physically reasonable modification of the ZM model. The SU(2) chiral sigma model [17] has the same defect as the ZM model. Its calculation using the parameter set

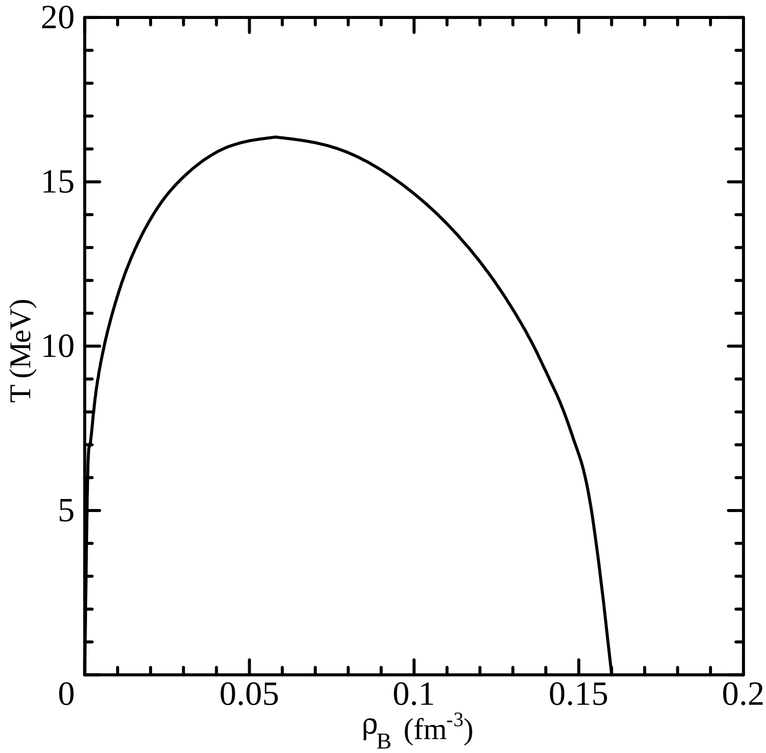

Figure 2: The liquid-gas phase coexistence curve in temperature-density plain.

II predicts $T_C = 16.8\,\mathrm{MeV}$ but $m^* = 0.85$.

The EZM model takes into account the effect of nucleon structure in the medium through the renormalized meson-baryon coupling constants. There are other models considering the similar effect based on the different pictures of nucleon structure from the naïve constituent quark model used in the EZM model. One is the quark-meson coupling (QMC) model and another is the chiral SU(3) quark mean-field model. The former [18] predicts $T_C = 17.7\,\mathrm{MeV}$ that is somewhat larger than the upper limit of the empirical value, while the latter [19] predicts $T_C = 15.82\,\mathrm{MeV}$ that lies just above its lower limit. It is however noted that the latter model has an ambiguity in determining the effective mass of a nucleon in the medium and another definition of it gives $T_C = 17.9\,\mathrm{MeV}$.

From the above comparisons between the EZM model and the other models, we can conclude that at present the EZM model is the only one to be able to reproduce both the nuclear matter saturation properties and the critical temperature simultaneously. It is therefore worthwhile to investigate the critical phenomena of nuclear matter in the EZM model, which have been observed in the recent experimental efforts [2,3,20]. Figure 2 shows the liquid-gas phase coexistence curve derived by the Maxwell construction of the mixed phase. Figure 3 shows the difference between the densities of the nuclear liquid and gas phases versus temperature on the coexistence curve. The circles are the results calculated at $T = 6.0,\ 7.0 \cdots 16.0,\ 16.1, 16.2$ and $16.3\,\mathrm{MeV}$. They can be fitted by the red and blue lines in the near region to and the far region from the critical point respectively.

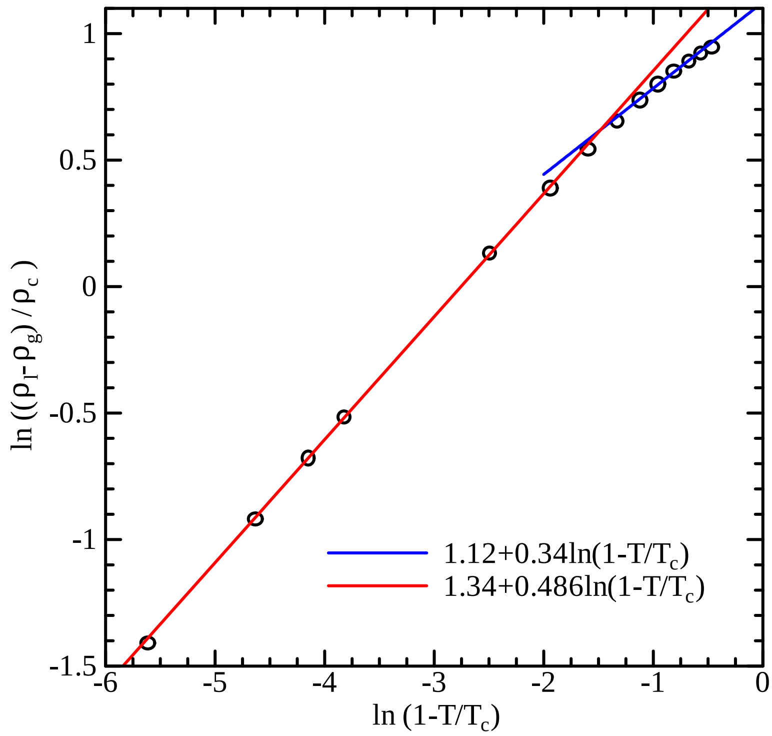

Figure 3: The difference between the densities of the nuclear liquid and gas phases versus temperature in log scale calculated on the phase coexistence curve in Fig. 2.

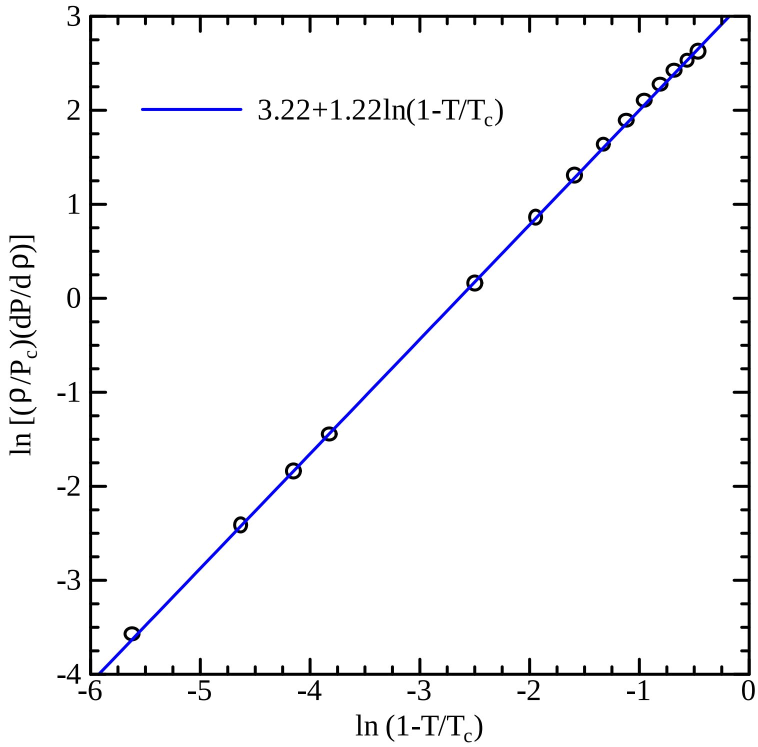

Figure 4: The inverse of compressibility versus temperature in log scale calculated on the liquid branch of the phase coexistence curve in Fig. 2 at the same temperatures as Fig. 3.

This confirms that the warm nuclear matter clearly exhibits the critical phenomenon. The inclination of the line is just the critical exponent β. Here we have to note [3] that in experiments the limit of the Coulomb instability prevents the nuclear system from reaching the critical point and so the empirical value of β is derived in the far region from the critical point. According to this fact, the result of Fig. 3 predicts $\beta = 0.34 \simeq 1/3$ from the blue line rather than $\beta = 0.49 \simeq 1/2$ from the red line. The value agrees well with those derived in Refs [2], [3] and [20], and with the universal value of the liquid-gas phase transition.

Moreover, the critical exponent γ (precisely γ') is investigated from the incompressibility κ,

$$\frac{1}{\kappa} \propto \frac{\rho_B}{P_C}\frac{\partial P}{\partial \rho_B} \propto \left(1 - \frac{T}{T_C}\right)^{\gamma}. \tag{16}$$

The circles in Fig. 4 show the results calculated on the liquid branch of the phase coexistence curve in Fig. 2 at the same temperatures as Fig. 3. In contrast to Fig. 3, both the results near and far from the critical point can be well fitted by the blue line only. The obtained critical exponent $\gamma = 1.22$ also agrees with the empirical value derived in Ref. [20] and the universal value of the liquid-gas phase transition. Although the critical temperatures in Refs. [2] and [20] have been derived for finite nuclei but not for nuclear matter, the critical exponents are independent of the temperature and so the comparisons of the exponents from the EZM model of nuclear matter with their experimental values of finite nuclei are physically meaningful.

We have studied the thermodynamics of warm nuclear matter below the saturation density in the EZM model. Although the model also produces the EOS like van der Waals one as the other microscopic EOSs, it is able to reproduce the empirical value of the critical temperature. It is important to reproduce the saturation and thermal properties simultaneously within the same theoretical framework. At present only the EZM model satisfies the condition. We have further investigated the critical phenomena and found the exponents β and γ to agree well with their universal values and empirical values derived in the recent experimental efforts. Because the critical phenomena have been confirmed in symmetric nuclear matter, the present investigation should be extended to asymmetric nuclear matter and strange hadronic matter in our future works.

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